passages in Part 2, § 3, which I here venture to quote, as they appear strongly to support the attribution to him of the approximation.

"from this greateſt phyſicall æquation [*i.e.* the arc whose circular measure is *e*], Kepler deduceth the particular æquation for every degree of anomalia eccentri, ſo finding thence anomalia media; but for to worke converſly he hath no direct way, neyther do J thinke any can be given: yet if inſtead of yᵉ number of ſeconds, he would vſe yᵉ ſine of yᵉ greateſt equation, and frõ thence find yᵉ ſine of each particular equation (which will meerly inſenſibly differ from his method) we may from anomalia media find anomalia eccentri viâ directâ. . . ."

Then follows a geometrical demonstration, ending with a precept for calculation, equivalent to equation (7), and corresponding to the Latin precept in *Op. Posth.* (p. 469, § 5), after which he proceeds :—

"The difference which is betwixt Keplers calculation and this is merely inobſervable, in ♂ it is but 10″ at moſt, in ☉ & ♀ not 1″, in ♄ & ♃ but 3″ or 4″, in ☿ it is 1′ 40″ ſometimes, wᶜʰ yet in his apparent place will not cauſe above 50″ difference wᶜʰ is iuſt nothing. . . . The cauſe of yᵉ difference is yᵗ Kepler reckons yᵉ length of yᵉ arch of yᵉ phyſicall equation, J the ſine, which increaſe not proportionably, & yet in ſmall arches with inſenſible difference. . . ."

Horrocks's estimate of the error is for the most part too small. Its approximate value from equation (8) is for ♂ 28″, ☉ 0″·2, ♄ 6″, and ☿ 5′.

In conclusion, I should like to express my appreciation of the helpful suggestions I have received from the Astronomer Royal and Dr. Jackson.

The Motions and the Distances of Spiral Nebulæ.

By Knut Lundmark, Ph.D. (Plates 18, 19.)

At present there are several different ways in which the distances of the spiral nebulæ can be estimated. The different methods to be used are properly divided into a direct and an indirect group. The former includes the rather few methods where no assumption is made concerning the physical nature of the spiral nebulæ. The latter embraces the many different methods where some assumption is made, for instance, that the spirals are agglomerations of stars, replicas of our stellar system, masses of gases or dust, or that individual objects observed in the spirals, such as novæ, cepheids, ordinary stars, and gaseous or dark nebulæ have, on the average, the same properties as the corresponding objects in the stellar system.

A. Direct Methods.

(a) *Measures of Parallaxes.*—I think it can justly be said that the attempts to measure the parallaxes of spiral nebulæ have shown that

these objects are out of reach for the present methods of direct parallax-measures.*

(b) *Proper Motions and Radial Velocities.*—The use of these data indicates a way of obtaining stellar distances which is fairly free from objections. The main difficulty in this method is the estimation of the influence of the different sources of error. In the case of spiral nebulæ this influence is very important, as the proper motions seem to be affected by errors of quite the same order of magnitude as the motions themselves.

Mainly on account of the enthusiastic and skilful work of V. M. Slipher we have now knowledge of 44 radial velocities of spiral nebulæ

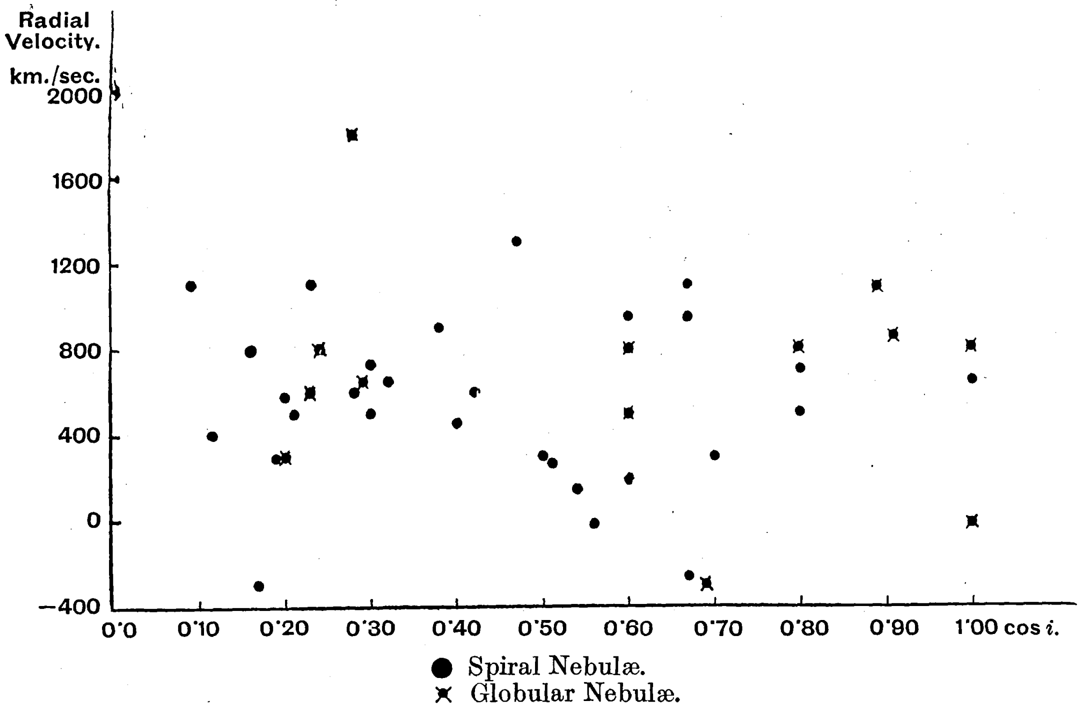


FIG. 1.—Relation between Inclination of Spiral Nebulæ (*i*) and Radial Velocity.

or related objects. In my paper in *M.N.*, **84**, 747, 1924, a few reasons were given for the opinion that the measured Doppler shifts of the lines are due either to motions in the non-relativistic sense or to motions and certain effects consequent to the general theory of relativity. Examining the existing set of radial velocities we find them exhibiting the following general features:—

As shown in fig. 1, there is not, as formerly has been supposed, a correlation between the inclination of the plane of the spirals and the radial velocity. Thus the spirals do not move like a discus thrown through space. Neither is there any definite correlation between the radial velocity and the galactic latitude. It is true that close to the galactic poles only high velocities are to be found, but, on the other hand, a velocity of a certain size does not tell anything for certain concerning the position of the object with regard to the galaxy.

* Mount Wilson Contr., No. 243, 1922.

A rather definite correlation is shown between apparent dimensions and radial velocity, in the sense that the smaller and presumably more distant spirals have the higher space-velocity. Another interesting feature is the relation between the radial velocities and the stage of nebular evolution. If we assume an evolutionary sequence in which the globular and elliptical objects without any traceable spiral structure are the youngest, and the irregular objects, tentatively termed the Magellanic Cloud type, are the oldest, we find :—

Class of Objects.	Mean Rad. Vel.	Number of Objects.
	km./sec.	
"Globular" nebulæ	727	11
"Early" spirals	647	18
"Late" spirals	396	5
Magellanic cloud-nebulæ *	263	4
Magellanic clouds	217	2

The most characteristic feature of the radial velocities of spirals is the presence of a very large Campbell shift of the same nature as is found in most classes of giant stars. At first it was thought that the shift shown in the spectra of spirals was constant, and the size of the K-term first determined by Wirtz and myself was fixed at +650 km./sec. Later on I reached the conclusion that the radial velocities are best represented by the expression :

$$X \cos \alpha \sin \delta + Y \sin \alpha \cos \delta + Z \sin \delta + k + lr + mr^2 - v = 0,$$

where X, Y, Z are the components of the Sun's motion relative to the spirals, k, l, and m are constants, v the observed radial velocity, and r the relative distance of a spiral derived from the apparent diameter and the total apparent magnitude, assuming the absolute dimensions to be the same.

The solutions showed that the value of m could not be very accurately determined from the 44 velocities. The final value was obtained by expressing the other unknowns as linear functions of m, and substituting these in the 44 equations of condition. Solving these again, we found the following value for the Campbell shift :

$$+513 + 10{\cdot}365r - 0{\cdot}047r^2 \text{ km./sec.}$$

The unit for r is the distance of the Andromeda nebula. It is thought that m, although inaccurately known, still expresses a real phenomenon. According to the above expression the shift reaches its maximum value, 2250 km./sec., at some 110 Andromeda units, which, according to results given later on, corresponds to a distance of 10^8 light-years. As the peculiar velocities of the spirals seems to be smaller than 800 km./sec., one would scarcely expect to find any radial velocity larger than 3000 km./sec. among the spirals.

* *N.G.C.* 6822, having a rad. vel. of +25 km./sec., should be included in this group. The following determinations of the apex were made before the velocity was published.

TABLE I.

	Material Used.	No. of Objects.	X.	Y.	Z.	V. in km./sec.	K. in km./sec.	A.	D.
1.	All objects	44	−132	+143	− 628	− 657	+ 765	313°	+7
2.	" " available in 1919	18	−218	−101	− 698	− 738	+ 659	25	+7
3.	Excluding N.G.C. 604 and 5195	42	−104	+135	− 620	− 643	+ 786	308	+7
4.	Irregular objects excluded	39	−113	+127	− 598	− 621	+ 786	311	+7
5.	Objects having spiral structure	26	−139	+294	− 466	− 568	+ 688	295	+5
6.	Globular nebulæ	13	−172	+480	−1092	−1203	+ 911	290	+6
7.	Irregular objects + Mag. clouds	5	−154	+358	− 10	− 389	+ 166	292	+ [illegible]
8.	Objects in the southern galactic hemisphere	12	−935	−921	−2421	−2754	+2387	44	+6[illegible]
9.	Objects in the northern galactic hemisphere	32	−645	+103	− 79	− 657	+ 146	352	+ [illegible]
10.	Objects having large diameters	13	−594	+ 64	− 104	− 606	+ 155	353	+1[illegible]
11.	Objects having small dimensions	31	− 60	+ 23	− 798	− 800	+ 933	339	+86
12.	Objects uniformly distributed	9	+ 36	−245	−1663	−1681	+1202	277	+81
13.	Nebulæ measured for rotation	5	−315	−387	− 400	− 646	+ 320	51	+39
14.	Globular clusters	18	− 91	+125	− 263	− 305	+ 31	306	+60
15.	Globular clusters	18	− 93	+ 94	− 274	− 304	0	314	+64
	All objects with								
16.	$K=+611+4{\cdot}75r-0{\cdot}0065r^2$	44	−132	+ 21	− 628	− 642	var.	351	+78
17.	$+10{\cdot}127r-0{\cdot}025r^2$	44	−325	+415	− 79	− 533	"	308	+79
18.	$513+10{\cdot}365r-0{\cdot}047r^2$	44	−121	+190	− 637	− 676	"	302	+71
19.	$625+3{\cdot}89r$	44	−136	− 6	− 626	− 641	"	3	+78
20.	$+6{\cdot}96r$	44	−356	+358	− 11	− 505	"	315	+ 1

Table I. gives the results from the different solutions for the apex. The following small table shows the frequency of the velocities obtained after substitution in the equations :—

Limits of Velocity. km./sec.	Number of Objects. *	**	***
−1000 to −1200	..	..	1
− 600 " − 800	1	..	1
− 400 " − 600	1	3	5
200 " − 400	10	8	6
0 " − 200	11	12	8
0 " + 200	10	9	12
+ 200 " + 400	7	5	4
+ 400 " + 600	3	7	5
+ 600 " + 800	1	..	2

* Campbell shift $=513+10{\cdot}365r-0{\cdot}047r^2$ km.

** " " $=625+3{\cdot}89r$.

*** " " $=765$.

The apex for the spirals seems now to be fairly well established. In fact, the velocity and the direction have changed very little from the time when only 18 objects, or scarcely more than a third of the present material, were available. The motion of our Sun is directed towards the galactic longitude $75° \pm 17°$, or nearly at right-angles to the direction (gal. long. $330° \pm 5°$) where modern conceptions of the universe place the centre of our galactic system. As I pointed out before,* this may indicate a rotation of our local cluster in the plane of the Milky Way around the Sagittarius region. Assuming circular orbits, the period of rotation comes out at 3.10^9 years, which is fairly close to the value 10^9 assigned by Charlier from theoretical considerations. On the other hand, if my hypothesis is correct, the mass of the stellar system within the orbit of the local cluster will be $10^{12}\odot$, which seems to be too high a value, as the number of visible stars is 10^{10}† and the total mass of the visible stellar universe something about $0{\cdot}8.10^{10}\odot$. Of course, dark stars and dark matter exist and increase that value, and, besides, the assumed distance for the central region (68,000 light years according to Shapley) may be somewhat too large. Still it seems likely that the apparent motion of the Sun may not be entirely attributed to a rotation of the local system.

As to the proper motions of the spirals our present knowledge is not very satisfactory. Many of the spiral nebulæ and the objects related to them have a very definite nucleus permitting accurate settings. Time after time measures have been performed and catalogues worked out giving accurate nebular positions. The oldest measures that can be used advantageously are from 1860–1865. The most accurate positions for this epoch were determined by Professor H. Schultz in Upsala, who measured 500 nebular objects. His results are very accurate and remarkably free from systematic errors.

Inasmuch as several astronomers during the last few years have advocated very moderate distances for spiral nebulæ, it occurred to me that it may be worth while to try to derive proper motions from the material existing. For 100 objects belonging to what may be called the family of spirals, about 1200 positions have been collected, using the following catalogues :—

Laguier, d'Arrest (I. and II.), Auwers, Schönfeld (I.–II.*a*), Harvard, Lord Rosse, Schmidt, Oppolzer, G. Rümker (I. and II.), Vogel (I. and II.), Schultz (I. and II.), Engelmann, Stephan (new reduction by Esmiol), Kempf, Porter (I. and II.), Weinek and Gruss, Ginzel, d'Engelhardt, Stone, L. Becker, Dreyer, Mönnichmeyer, Spitaler, Bigourdan, Merecki, Keeler, Wolf, Lorentz, Rheinmuth (I. and II.), Kobold, Wirtz, Winnecke (new reduction by E. Becker), Bredichin, Peters, and occasionally positions given in other sources, for instance, some of the star catalogues.

A number of the older positions have been improved by Wirtz or myself using new positions for the comparison stars.

The nebular positions have been corrected for systematic errors and weights assigned, whereby a compromise was made according to

* *M.N.*, **84**, 747, 1924.
† *Nature*, No. 2903, 1925 June 20.

the following factors : (1) the number of observations and the internal agreement ; (2) the brightness and the accuracy of the positions of the comparison stars ; (3) the method of observation (micrometer, heliometer, meridian instruments, photography) ; (4) the instrument used ; (5) the size of the systematic errors.

The weights run from 0–6, the last given only to a few modern photographic measures. Of the old positions those of Schultz have, as a rule, the weight of 4.

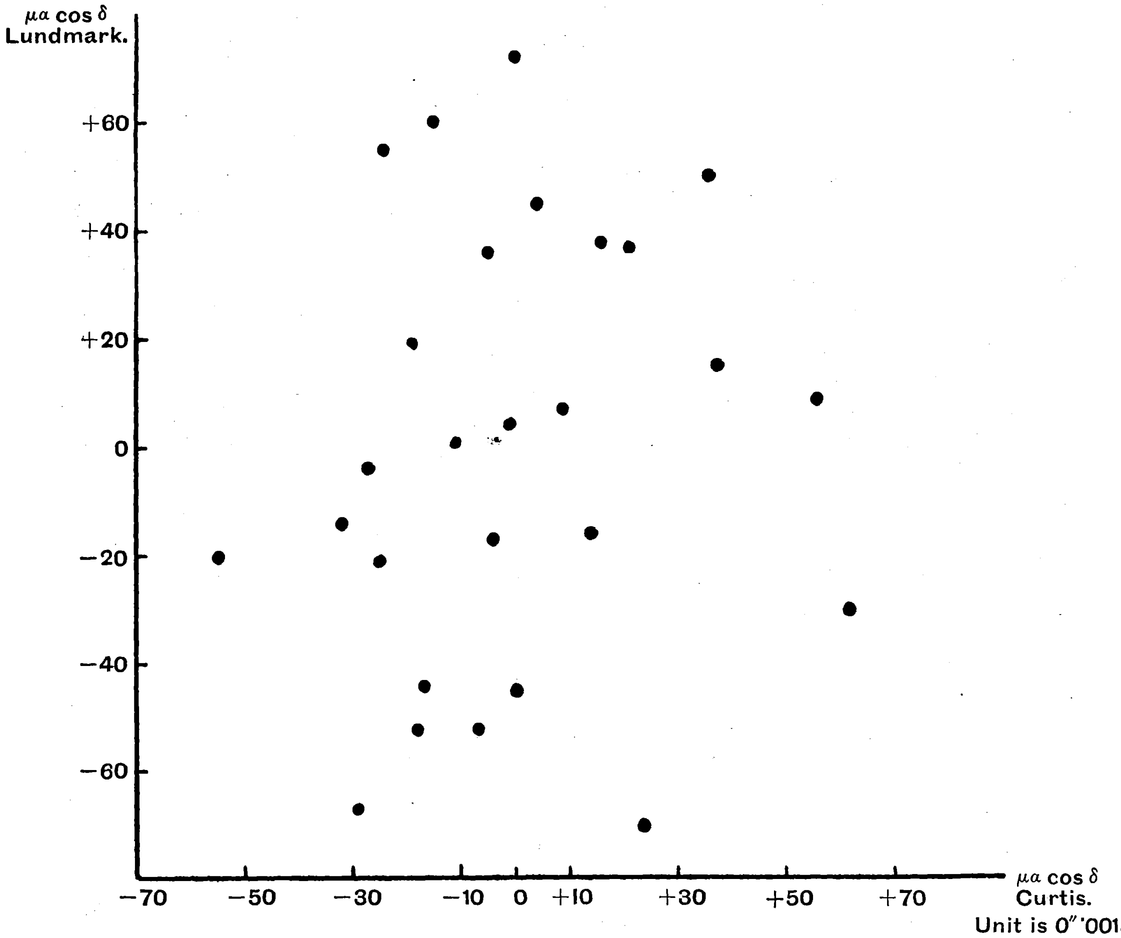


Fig. 2.—Comparison between the Right-Ascension Components of the Proper Motion of Spiral Nebulæ as derived by Curtis and by Lundmark.

Through the efficient aid of Mr. K. Littmarck, M.A., least square solutions have been carried through for 83 objects. The average values of the proper motions are : $\bar{\mu}_\alpha = 0^s{\cdot}0028$; $\bar{\mu}_\delta = 0''{\cdot}031$, and the algebraic means, $\mu_\alpha = -0^s{\cdot}0000$, $\mu_\delta = +0''{\cdot}013$; the average mean error is $\epsilon(\alpha) = \pm 0^s{\cdot}0037$; $\epsilon(\delta) = \pm 0''{\cdot}039$. The average value of the total proper motions is $0''{\cdot}045$. The motions derived are evidently of the same order of magnitude as their errors.

Computing the apex of the Sun's way from my results I have found : $A = 19^{h}{\cdot}2$; $D = -69°$; $q = 0''{\cdot}0135$. The poor agreement with the apices derived from radial velocities shows that some systematic error must have crept into the material.

A comparison of my proper motions with those of Curtis will strengthen this conclusion. Measuring Crossley plates with a mean interval of 13·85 years, Curtis found, as is well known, a mean motion for 66 spirals of 0″·033. Using his values according to Trümpler's rediscussion we find the apex: $A=4^{h}\cdot 0$; $D=-59°$; $q=0''\cdot 0147$.

Twenty-nine objects have been determined both by Curtis and by myself. Comparing them we find no systematic difference in declination, but for the right ascension components there is the relation:

$$\mu_{L}-0''\cdot 013=\mu_{C},$$

where the subscripts denote the motions of the observers. In the solution made for the apex, using my material, the μ_{L} have accordingly been reduced to Curtis' system.

As will be seen in the graphs, there is practically no relation between the proper motions of Curtis and myself. The correlation-coefficients are, in fact, illusory: $r=-0\cdot 023\pm 0\cdot 192$ (for the right ascensions), and $r=+0\cdot 147\pm 0\cdot 181$ for the declinations).

Assuming the real motions distributed as accidental errors, we have the following relations:

$$(n-1)\ (\mu^{2}+\epsilon_{L}{}^{2})=\Sigma\mu_{L}{}^{2}$$
$$(n-1)\ (\mu_{2}+\epsilon_{C}{}^{2})=\Sigma\mu_{C}{}^{2}$$
$$(n-1)\ (\epsilon_{L}{}^{2}+\epsilon_{C}{}^{2})=\Sigma(\mu_{L}-\mu_{C})^{2},$$

where ϵ are mean errors, μ the mean of the squares of the individual motions, and n the number of objects in common. From the material we get the following results:

$$\epsilon_{L}(\alpha)=\pm 0''\cdot 044 \qquad \epsilon_{L}(\delta)=\pm 0''\cdot 033$$
$$\epsilon_{C}(\alpha)=\pm 0''\cdot 033 \qquad \epsilon_{C}(\delta)=\pm 0''\cdot 024$$
$$\mu_{\alpha}=\pm 0''\cdot 018 \qquad \mu_{\delta}=\pm 0''\cdot 016.$$

Thus there should be an equal chance that the real proper motions in right ascension are inside the limits $\pm 0''\cdot 012$. The corresponding limits for the declination components are $\pm 0''\cdot 011$. It is not likely that the motions actually are as large as 0″·01, but the computation shows that Curtis' motions as well as my own are probably spurious and mainly due to the errors of the methods.

Another set of proper motions of nebulæ is obtained by van Maanen as a by-product from his measures of internal nebular motions. The outcome of these valuable proper motions has been discussed in the *Observatory*, **57**, 279, 1924. From the objects in common with Curtis, we find:

$$\mu_{C}(\alpha)-0''\cdot 006=\mu_{vM}(\alpha)$$
$$\mu_{C}(\delta)-0''\cdot 025=\mu_{vM}(\delta).$$

Although the systematic differences are uncertain on account of the few objects in common, a new solution for the apex has been made for Curtis' proper motions reduced to the system of van Maanen. The results are given in the table below.

Now, it may very well be possible that the proper motions derived by me are affected by an error in the adopted value of the constant

of precession. The spiral nebulæ in any case are rather distant objects, but the precession is computed using values derived from neighbouring stars. In order to test the question the following equations were formed :

$$x \sin \alpha - y \cos \alpha + \Delta n \sin \alpha \sin \delta + \Delta\mu_0 \cos \delta = \mu_\alpha \cos \delta$$
$$x \cos \alpha \sin \delta + y \sin \alpha \sin \delta - z \cos \delta + \Delta n \cos \alpha = \mu_\delta,$$

where x, y, z are the components of the Sun's motion, $\Delta\mu_0$ a constant

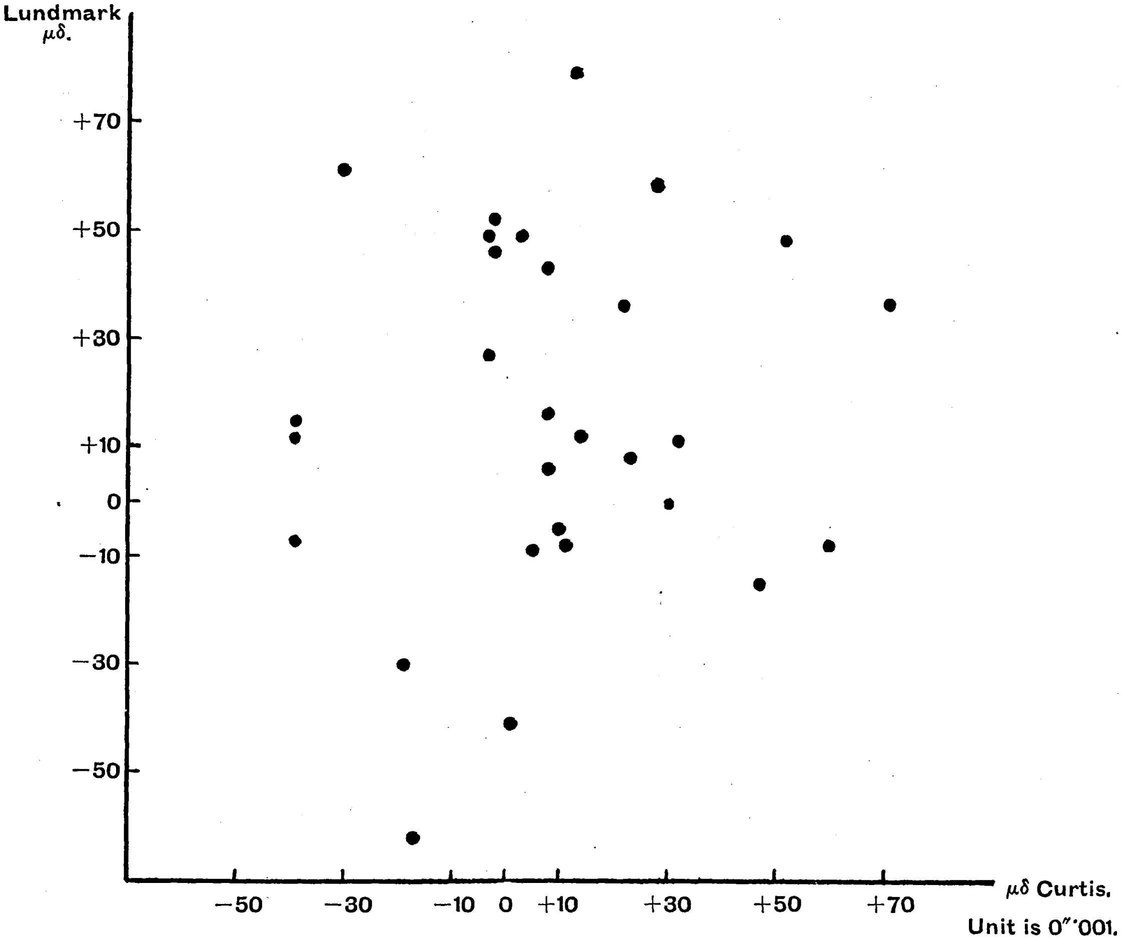


Fig. 3.—Comparison between the Declination Components of Proper Motion as derived by Curtis and by Lundmark.

correction, and Δn the correction to the precessional constant, n, the annual motion of the celestial pole towards the equinox.

The value of $\Delta\mu_0$ is $= -0''\cdot0096$ and the value of Δn is $= -0''\cdot015$. The latter value should be the correction to the constant of precession used, which is Struve-Peters for 1900.* It seems to me rather doubtful if the correction is real, as Boss derives a positive correction and so does Eichelberger. Certainly, it will be desirable to wait for a new determination of Δn when more material has been collected.

* The correction to Newcomb's constant would be $-0''\cdot012$, which is very improbable. The correction found must certainly be attributed to other causes than an error in the precession constant.

As to the motions of Curtis or van Maanen, they are not affected by any influence of precession.

It is interesting to note that the apex derived from the above equations is rather close to the antapex of ordinary stars, and this is also the case with the apex computed from Curtis' uncorrected motions. It can easily be shown that if what we measure is not the motions of the nebulæ, but the reflected motions of the comparison stars, this will give us the co-ordinates of the antapex when we make a solution for the apex.

In the following table we give a summary of the determinations of apices from the proper motions of spirals:

Material.	No. of Objects.	*x*.	*y*.	*z*.	*q*.	A.	D.
		$''$	$''$	$''$	$''$	h	°
Curtis' motions	66	+0·0056	+0·0038	−0·0132	0·0147	4·0	−59
,, ,, corrected	66	+0·0067	+0·0026	+0·0154	0·0170	1·4	+65
Lundmark's motions corrected	83	+0·0016	−0·0048	−0·0145	0·0153	19·2	−69
Lundmark's motions corrected and a correction to the precession constant supposed	83	+0·0108	+0·0062	−0·0109	0·0165	2·0	−41
Van Maanen's motion	7	+0·0035	+0·0025	+0·0041	0·059	2·4	+44
Van Maanen's motion rediscussed by Smart	7	+0·0030	+0·0014	+0·0064	0·0074	1·7	+63
Wirtz's motions *	98	+0·0089	+0·0248	+0·0178	0·0320	4·7	+34

Although the proper motions are not as yet known, still we can get an idea of the mean parallax of spiral nebulæ. Our discussion shows that the real motions of spirals are well below 0″·01. Using the values of *q* as representing an upper limit for the real *q* and combining with the determinations of the apex from radial velocities, we find a minimum distance of 30,000 light-years for the spiral nebulæ.

To this will only be added that the dropping of the well-established K-term will give a mean distance of the same order of magnitude, because the assumption K=0 leads to a value for the velocity of the Sun of the same size or even larger than the one adopted here.

(c) *Internal Motions.*—If the radial velocity can be measured for different points in the same spiral, and measures of direct photographs have given the apparent size of internal motions, fairly reliable distances can be derived from such data. As is well known, Dr. van Maanen has measured the internal motions of 7 spiral nebulæ. During a stay at the Mount Wilson Observatory, at the suggestion of Dr. Adams and Dr. van Maanen, I measured the same plates of Messier 33 as the latter. The same instrument, the new large stereocomparator with a "blink" arrangement, was used for the measures. Altogether 400 points well distributed in the spiral arms, referred to 24 comparison stars, were measured. The measures were made at the same time as van Maanen

* Wirtz's values, which have not been published in detail, evidently are not computed from all the existing material for every object but from three or four positions in the average. Regarding details, see *A.N.*, **204**, 26, as well as other papers by Wirtz, giving the results of his extensive investigations concerning proper motions of different groups of nebulæ.

performed his work, but the reduction of my set was carried out after my return to Upsala. The detailed comparison of the two series of measures will be given later on, but the following general results may be mentioned.

The proper motion of the nebula as a whole is found to be: $\mu_\alpha = -0''{\cdot}0015$; $\mu_\delta = -0''{\cdot}0050$, whereas van Maanen has $\mu_\alpha = +0''{\cdot}0034$; $\mu_\delta = -0''{\cdot}0044$. Resolving my motions into rotational and radial components, the mean values are found to be: $\bar{\mu}_{\text{rot}} = +0''{\cdot}0016$; $\bar{\mu}_{\text{rad}} = -0''{\cdot}0056$. The corresponding values from van Maanen's measures are: $+0''{\cdot}0196$ and $-0''{\cdot}0029$ respectively. The dispersions around the means are: $\sigma_{\text{rot}} = \pm 0''{\cdot}0065$; $\sigma_{\text{rad}} = \pm 0''{\cdot}0091$ (Lundmark); $\sigma_{\text{rot}} = \pm 0''{\cdot}0080$; $\sigma_{\text{rad}} = \pm 0''{\cdot}0096$ (van Maanen).

The rotational motion found by me can be represented by the linear formula:

$$\mu_{\text{rot}} = -0''{\cdot}00084 + 0''{\cdot}00032\rho,$$

where ρ is the distance from the centre of the nebula in minutes of arc.

A similar solution of van Maanen's components gave:

$$\mu_{\text{rot}} = +0''{\cdot}01063 + 0''{\cdot}00119\rho.$$

As the mean distance from the centre is $7'{\cdot}61$, the mean period of rotation will be $2{\cdot}8.10^6$ years and $0{\cdot}23.10^6$ years respectively.

If we use van Maanen's results, comparing them with the rotation of the spirals N.G.C. 224, 3031, and 4594, spectroscopically measured, we find distances ranging from 10,000 to 40,000 light-years. My measures suggest distances between the values 40,000–160,000 light-years. On account of the few cases investigated as yet, considerable uncertainty is involved in the application of the method.

The following formula is easily deduced:

$$M = [3{\cdot}71856]\mu_{\text{rot}}^2 \tan \rho \,.\, \pi^{-3},\text{*}$$

where M is the mass within the distance ρ from the nucleus of the nebula expressed in minutes of arc, and π the parallax of the nebula. Assuming a parallax of $0''{\cdot}000003$ we find the following values for M:

$1{\cdot}6 \,.\, 10^{14}\odot$ (van Maanen)
$1{\cdot}1 \,.\, 10^{12}\odot$ (Lundmark).

(d) *Absorption in Space.*—Another direct way for estimating the spiral distances could be obtained from determinations of the amount of selective absorption in space. The effective wave-lengths of spirals as measured in Upsala by Lindblad and myself † permitted a computation of the spectral types by using the relation between λ_{eff} and spectra as established from investigations of stars. The graph in fig. 6 shows the fair agreement between the computed and classified spectra. Now, the former depend entirely on the energy distribution in the continuous spectrum, whereas the latter are obtained from scrutinising the balance between certain groups of absorption lines. Thus a selective absorption, if present, will make the computed types later than the observed ones. The mean spectral type computed is G4·5 and the mean observed G1·7. Evaluating the spectral types in colour-index we find

* Publ. *A.S.P.*, **34**, 108, 1922. † Aph., J., **46**, 206, 1917; **50**, 376, 1919.

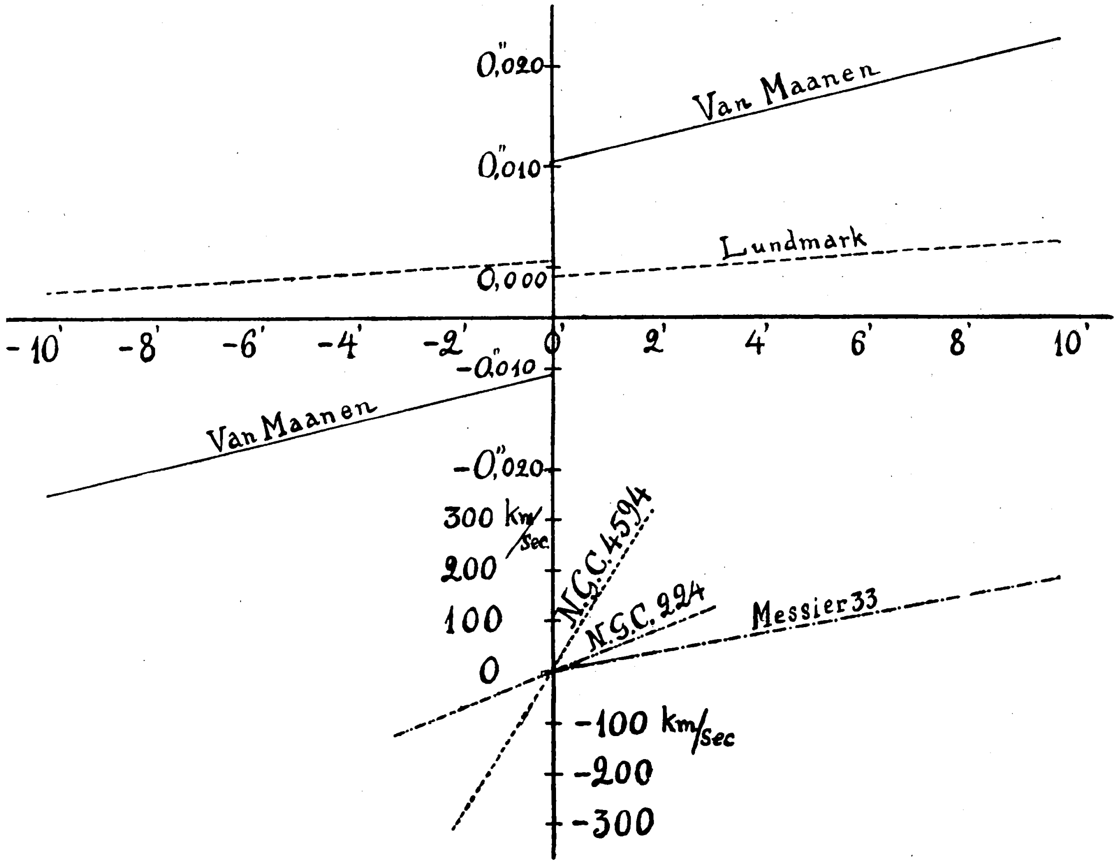


FIG. 4.—The Rotation of Spirals spectroscopically and directly measured. Abscissæ are angular distance from the centre of the Spirals.

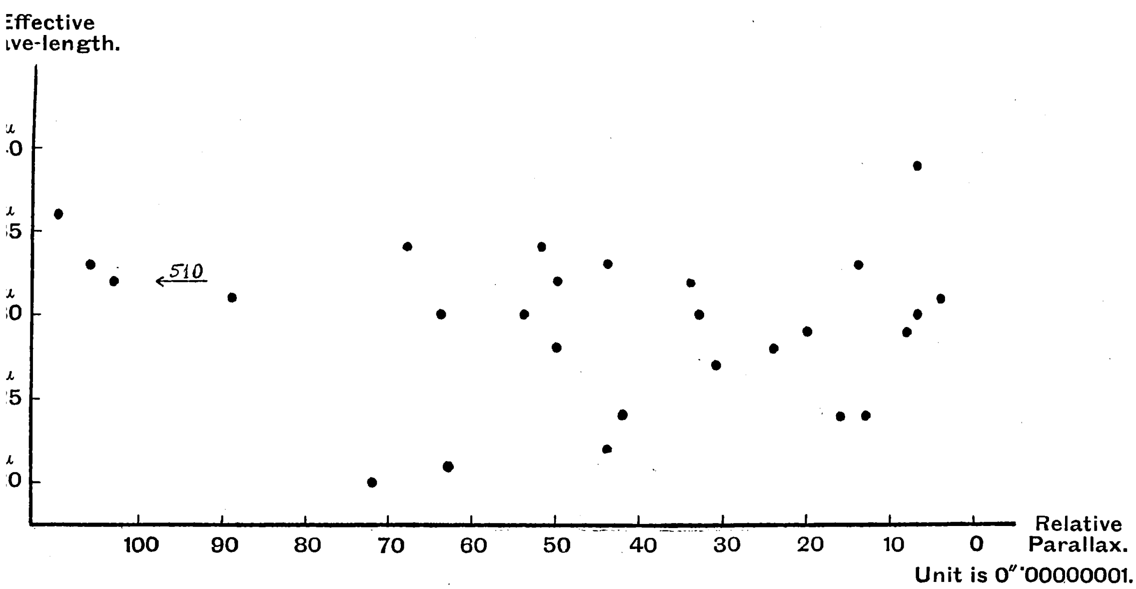


FIG. 5.—Effective Wave-Lengths of Spiral Nebulæ as plotted against relative Parallaxes of the Objects.

the difference to be $+0^{m}\cdot12$. If we had a final determination of the coefficient of the selective absorption we could derive *a maximum value* of the mean distance of the spirals. As the value of Shapley is a limiting value, showing that the coefficient is smaller than $0^{m}\cdot000002$ per parsec, I prefer for the present to go the other way and make an independent estimate of this coefficient. The mean distance of the objects used is 20 Andromeda-units, which should correspond to a distance of 20 million light-years. To be on the safe side, I assume 10

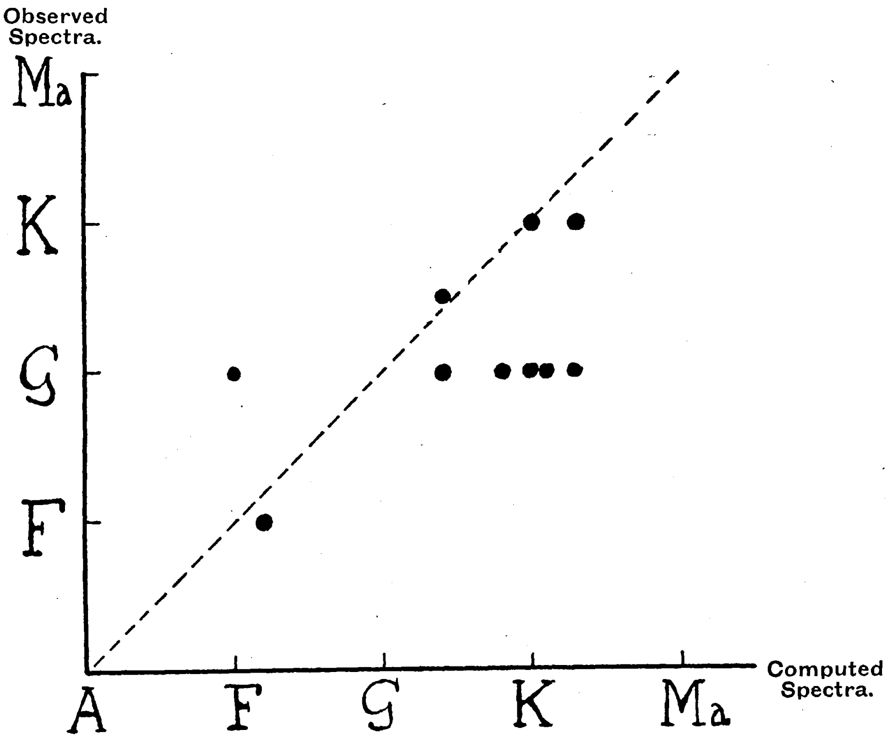


FIG. 6.—Comparison between Spectra of Spirals computed from effective Wave-Lengths and directly classified.

million light-years as representing the mean distance, and thus we find the following value of the coefficient of selective absorption :

$$\leq 0^{m}\cdot000000012 \text{ per light-year.}$$

The interesting question arises: Is there any general absorption among the spiral nebulæ? The graph in fig. 7 shows a comparison between the photometric measures of the surface intensity of spiral nebulæ measured by Wirtz and the relative parallaxes computed by me. As pointed out by Wirtz himself, if there are not systematic errors making the smaller spirals measured fainter than the larger ones, the only possible explanation will be that we are dealing with a general absorption or rather extinction affecting equally all wave-lengths. From the material used in the graph I have derived a coefficient of the absorption amounting to $0^{m}\cdot02$ per Andromeda-unit. Assuming as before 10^{6} light-years for that unit, we have :

$$\text{general absorption} = 0^{m}\cdot00000002 \text{ per light-year.}$$

At the suggestion of Professor Hertzsprung the density of meteoric matter has been computed from this coefficient. Using the formula: *

$$\text{extinction in magnitudes} = \frac{p \cdot m}{d \cdot s},$$

where p is a constant $=0{\cdot}407$, m the average meteoric matter in unit space, and d the mean diameter of meteors, and s their mean density, we have adopted the following values: $d=6{\cdot}25 \cdot 10^{-3}$ grams, † $s=7{\cdot}9$

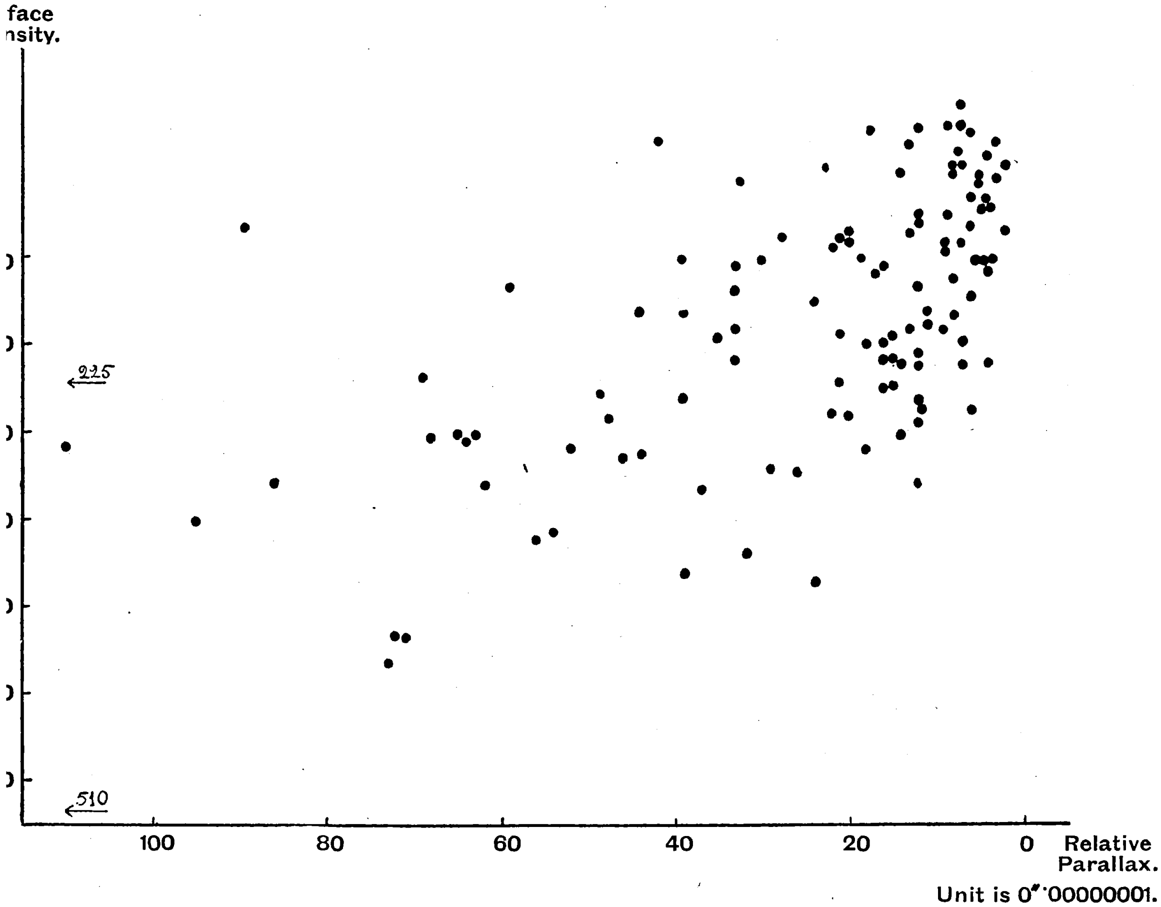


FIG. 7.—Relation between the Surface Intensity of Spiral Nebulæ and the relative Parallax.

(iron). Thus we find that within a cube having a side of one astronomical unit there should be:

$$0{\cdot}6 \cdot 10^{16} \text{ meteors.}$$

According to Hoffmeister's investigations,‡ founded on actual observations of the frequency of meteors, the number in the same cube is:

$$10^{16} \text{ meteors.}$$

The excellent agreement strengthens the conclusion that Wirtz's observations really show the presence of a general absorption in space.

* *Proc. of the Nat. Academy of Sc. Washington*, **8**, 115, 1922.
† *Proc. of the R.S.*, **102**, 411, 1922.
‡ *Ergänzungshefte zu den Astr. Nachr.*, Bd. 4, Nr. 5, 1922.

B. Indirect Methods.

(a) *Novæ in Spirals.*—One of the most interesting characteristics of the spiral nebulæ is perhaps that they seem to possess the necessary conditions for producing the so-called Novæ, as also is the case with certain regions in our stellar system, as a rule situated close to the galactic plane. The first Nova discovered in a spiral nebula was the one named S Andromedæ which appeared in 1885 close to the nucleus of the great nebula in Andromeda. As this spiral covers an area of 1/42,000 of the sky, the probability is rather small that we should have seen the star only by chance in direction towards the nebula. At the time of discovery several astronomers thought this was the case, and in spite of a few subsequent appearances of Novæ in spirals or related objects (N.G.C. 5253, 6093, 4147, and 5457–58) the importance of the phenomenon was not realised till later on. In the year 1917 it was concluded from the discoveries of several American astronomers that the appearance of Novæ in spirals must be a rather common event. Up to the present 22 Novæ have been observed in the Andromeda nebula.* The probability that these objects are seen only by chance in the same direction as the nebula is of the order of 4.10^{-88}, and thus there is not the slightest doubt that the stars in question actually are situated in the spiral. If any other proof should be thought necessary, it may be pointed out that there is a distinct crowding of Novæ around the central parts of the Andromeda nebula. Besides, as Novæ are not known in the stellar regions in the neighbourhood of the nebula, the discoveries are not favoured by a more thorough scrutiny of this spiral than of other parts of the sky.

Curtis † seems to have been the first to realise the cosmical importance of the occurrence of Novæ in spiral nebulæ. By assuming equality in absolute magnitude for galactic and spiral Novæ he concluded that the latter, being apparently ten magnitudes fainter, are about one hundred times as far away as are the former. Estimating the mean distance of galactic Novæ to be 100,000 light-years, he reached the conclusion that the spirals are galactic systems in size comparable with our own, and *that the closest spirals are millions of light-years away.* In 1919 I discussed the different ways of estimating the distances of spirals.‡ In this work the mean absolute magnitude of four galactic Novæ was derived as −4. From that value the parallax of the Andromeda nebula was computed to be 0″·0000051 (650,000 light-years) and the dimensions 23,000 light-years. That value has been accepted or confirmed later on by Curtis, § Luplau-Janssen, Haarh, ‖ Doig, ¶ and others on the basis of Novæ, and by Oepik ** and Hubble †† on the basis of rotational evidence and appearance of Cepheids respectively.

* In fact, 24 more Novæ have been discovered by Mr. Hubble, but particulars have not been published as yet.

† *Journal of the Washington Acad. of Sc.*, **9**, No. 8, 1919; *L.O.B.*, 300.

‡ *Astr. Nachr.*, **209**, 369, 1919; *Kungl. Sv. Vet. Akad. Handl.*, Bd. 60, 1920.

§ *Rivista di Scienza*, **35**, No. 1, 1924.

‖ *Astr. Nachr.*, **215**, 285, 1922.

¶ *Journ. of the B.A.A.*, **32**, 138, 1922.

** *Ap. Journ.*, **55**, 409, 1922.

†† *Observatory*, **48**, 139, 1925.

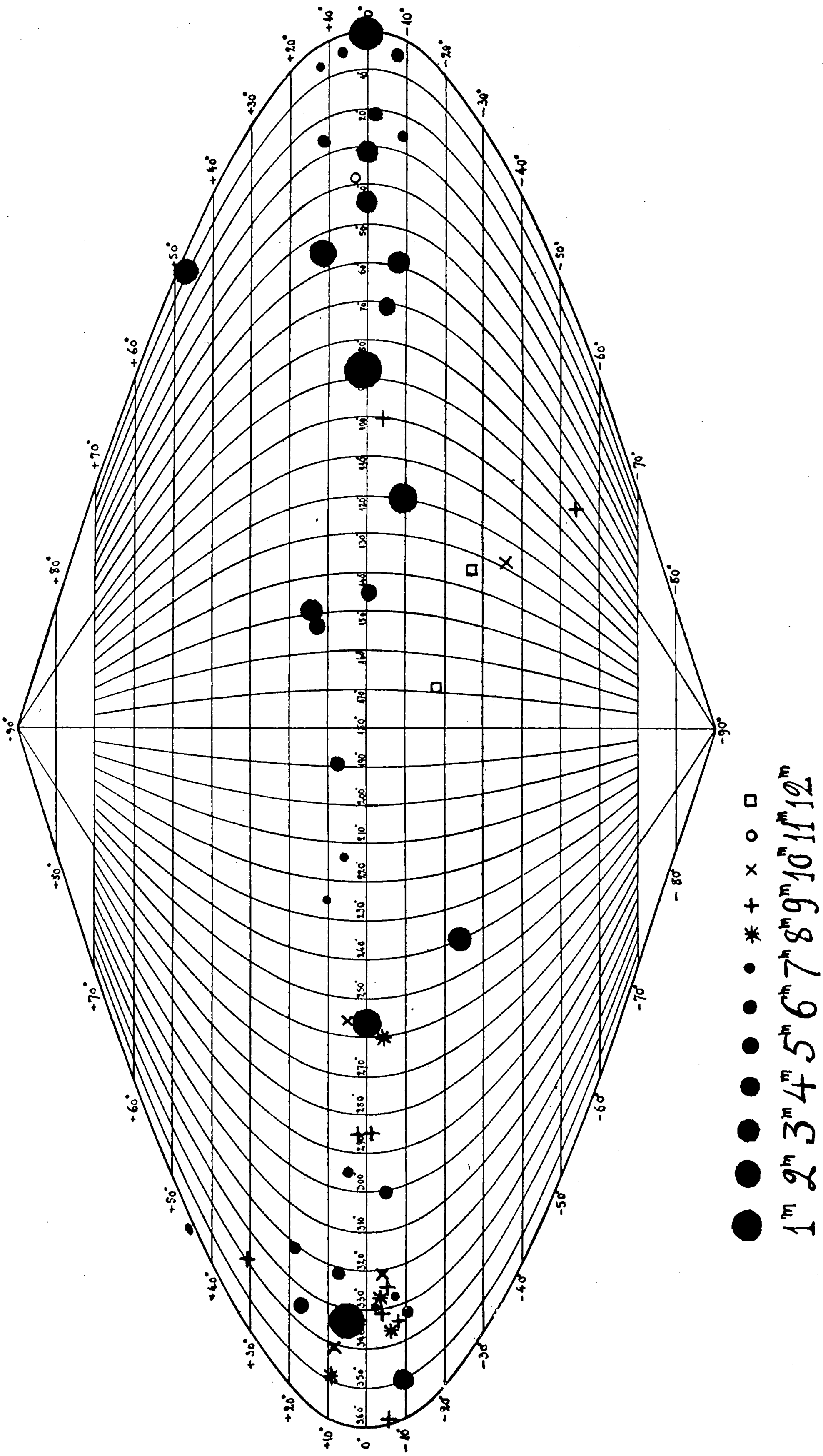


Fig. 8.—Galactic Distribution of Novæ (those in Spiral Nebulæ are excluded).

During the following years the absolute magnitudes of the galactic Novæ have been determined by me, using different methods. In the following I give a brief summary of the results.

a. Direct Measured Parallaxes or Determinations of Hypothetical Parallaxes for Individual Objects.—The following table gives a summary of the results:

Object.	Mean of direct Measured Parallaxes.	Proper Motion Parallax.	Spectral. Parallax	Hypothetical Parallax.	Adopted Mean.	Absolute Magnitude. At Maximum.	At Minimum.
Nova Persei 2	0″·0082	0″·0070	..	0″·0093	0″·0082	−5·4	+8 to +1[illegible]
„ Aquilæ 3	0·0025	0·0026	0·0022	0·0040	0·0031	−8·8	+3·0
„ T Coronæ	..	0·0016	0·0015	..	0·0016	−7·1	+0·6
„ Geminorum 2	0·005	0·0080	..	..	0·0060	−2·8	+7·0
„ Lacertæ	0·006	0·0003	..	..	0·0040	−1·1	+8·4
„ RT Serpentis	..	..	0·003	..	0·0030	?	+7·4
„ T Scorpii	..	..	..	0·00006	0·00006	−9·1	?
„ P Cygni	−0·013	0·0058	0·0032	..	0·0021	−4·6	−2·7
„ T Aurigæ	..	0·0015	..	..	0·0015	−4·6	+5·0
„ η Carinæ	..	0·0017	..	0·0015	0·0016	−7·0	+0·6
„ Cygni 1920	0·010						
„ Ophiuchi No. 4	−0·005						

Remarks.—*Nova Persei*—It will be of interest to see how the agreement is between different measures of the parallaxes of this star:

$\pi_{abs.}$	$\pi_{abs.\ red.}$	Weight.	Authority.
+0″·026±0″·010	+0″·032	0·8	Bergstrand.
+0·020±0·020	+0·025	0·3	Hassenstein.
+0·005±0·004	+0·005	2·0	Küstner.
−0·012±0·023	−0·001	0·5	Chase.
−0·010	−0·010	0·3	Hartwig.
+0·027±0·009	+0·027	0·3	Rambaut.
+0·007±0·004	+0·006	2·0	Van Maanen.

The second column gives the parallax reduced to absolute parallax and to Van Maanen's system.

The parallaxes in the upper table headed "proper motion parallax" are derived from the relation between the quantities m_π and m_μ as established for different spectral classes.* The spectral parallaxes have been derived from the knowledge of the spectrum when the star was in its normal condition.†

Various hypothetical parallaxes have been derived for this star using the reflection theory, that is the assumption that the outburst was reflected from the dark matter around the star. Of the different values we have selected the one given by Turner.‡ The close agreement between the values must be taken as a strong argument in favour of the reality of the derived parallax.

* *Lick Obs. Bull.*, No. 339, 345, 1922.
† *Publ. A.S.P.*, **34**, 147, 1922.
‡ *M.N.*, **79**, 23, 1918.

Nova Aquilæ No. 3.—The following parallaxes have been measured :—

	$\pi_{abs.}$	$\pi_{abs.\ red.}$	Weight.	Authority.
Photographic determinations	$+0''.019 \pm 0''.006$	$0''.018$	1·5	Van Maanen.
	-0.001 ± 0.008	0·007	1·2	Daniel.
	-0.011 ± 0.005	−0·007	1·0	Alden.
	-0.008 ± 0.007	−0·004	1·0	Olivier.
	-0.004 ± 0.013	−0·004	0·8	Fox.
	-0.014 ± 0.006	−0·006	1·0	Lee.
From Meridian observations	$+0.071 \pm 0.013$		0·3	Courvoisier.
	$+0.197 \pm 0.050$	(−0·041)	0·2	Philippot.
	$+0.064 \pm 0.072$		0·2	Delporte.
	$+0.160 \pm$		0·1	Trousset.

In *Publ. A.S.P.*, **34**, 208, the value of Olivier was erroneously quoted by me, and the parallax of Alden had been overlooked.

The meridian observations, which are evidently influenced by systematic errors, could just as well be excluded in forming the mean. As shown by Henroteau, the large positive parallax by Philippot can be reduced to a negative one when the proper motion is introduced into the solution. As the actual proper motion is certainly very small, the large value resulting from Philippot's material has to be interpreted rather as a magnitude equation varying with the time.

The hypothetical parallax value was derived by me by comparing the expansion in angular and linear speed of the nebulous disk.*

Nova T Coronæ.—My spectrograms from 1921 showed the star to be an Mb giant, from which fact the spectral parallax was derived.

Nova Geminorum No. 2.—Two directly measured parallaxes have been published, viz., $+0''.006 \pm 0''.008$ by Slocum, and $-0''.019 \pm 0''.013$ by Miller.

Nova Lacertæ.—The measured parallaxes are, $+0''.007 \pm 0''.012$ by Slocum and Mitchell, and $-0''.005 \pm 0''.020$ by Balanowsky.

Nova RT Serpentis.—The parallax is, in fact, a "spectroscopic" one, taken from Mount Wilson Contr., No. 199.

Nova T Scorpii.—This Nova is of unusual interest as having appeared close to the centre of the globular cluster Messier 80. As pointed out before, it probably is a *bona fide* Nova, really situated in the cluster. Using Shapley's parallax for M80, we obtain the absolute maximum magnitude −9·5, and using my value −8·7.

Nova P Cygni.—Parallax measured by Slocum and Mitchell, $-0''.021 \pm 0''.016$.

Nova η Carinæ.—For the nebula itself a parallax of $+0''.0015$ has been deduced in *Publ. A.S.P.*, **34**, 40, 1922. I have assumed the Nova to be connected with the nebula, which gets confirmation from the value of the proper-motion parallax.

Nova Cygni 1920.—Two trigonometric values for the parallax have been published, viz., $0''.020 \pm 0''.006$ (Daniel),† and $0''.003 \pm 0''.005$ (Van Maanen).‡

Nova Ophiuchi No. 4.—Van Maanen has measured the parallax $-0''.005 \pm 0''.008$.§

Using all the measured parallaxes alone (positive and negative values), the mean absolute maximum magnitude is found to be −7·0. If we combine the different hypothetical parallaxes together with the measured we get −6·2 for the same quantity.

β. Proper Motions and Radial Velocities.—From a discussion of the 11 proper motions given in *Publ. A.S.P.*, **34**, 207, and the 8 radial velocities quoted in the same paper, a mean absolute magnitude of −7·7 at maximum has been derived. One may think that the proper

* *Publ. A.S.P.*, **33**, 218, 1921.
† *Pop. Astr.*, **32**, 221, 1924.
‡ *A.J.*, **36**, 46, 1925.
§ *Ibid.*

motions are too small and inaccurate to allow a derivation of mean parallax. It may be very instructive to compute the values of the quantities $m_\pi = m+5+5 \log \pi$ and $m_\mu = m+5+5 \log \mu$ from the Nova material. In fig. 9 the relation between the absolute magnitude and the absolute proper motion magnitude is given from the best parallaxes and proper motions we have for the present. The fact that the same quantities for Novæ at minimum light fall rather nicely into the diagram suggests that the parallaxes of Novæ as well as their proper motions are free from systematic errors of importance.

γ. *Dip Effect of the Novæ.*—The Sun is situated some 200 light-years to the north of the galactic plane. The Milky Way star clouds have a mean galactic latitude of $-0°{\cdot}98$ and the Novæ have a " dip " of $-1°{\cdot}02$, showing the intimate relationship between these and the Milky Way structure. The mean distance that results from the dip gives a mean absolute magnitude at maximum of $-6{\cdot}5$.

δ. *The concentration of the Novæ in the Sagittarius region* gives another means for deriving the mean absolute magnitude. If we presume that the 16 Novæ found in the region actually belong to the central parts of the stellar system, we get, adopting Shapley's value of 68,000 light-years for the distance, the m_π(max.)$=-9{\cdot}5$. Correcting the value according to Wilson's determinations of the zero point in the period-luminosity relation of Cepheids we obtain m_π(max.)$=-8{\cdot}7$.

ε. *Comparison between the galactic distribution of Novæ and other classes of Milky Way objects* gives another possibility of estimating m_π for Novæ. Suppose, for instance, the mean of the co-ordinates of Novæ and B stars to be the same, or that the dispersions in the same co-ordinate are equal. This gives m_π (max.)$=-4{\cdot}2$.

Summing up the results from the different methods we have :

Method.	m_π (max.).	Weight.
α	$-6{\cdot}2$	3·0
β	$-6{\cdot}5$	1·0
γ	$-8{\cdot}2$	1·0
δ	$-7{\cdot}7$	3·0
ε	$-4{\cdot}2$	0·5
Unweighted mean	$-6{\cdot}6 \pm 0{\cdot}70$	
Weighted mean	$-6{\cdot}9 \pm 0{\cdot}66$	

The latter value will be adopted in this paper.

We see that the Novæ at their maximum are among the absolutely brightest stars known in the Galaxy. If the same is the case with the Novæ in spirals it may explain why *Cepheids* or other variable stars have not been discovered as yet.* These stars being absolutely fainter than the Novæ will only reach 18^m or 19^m at their maximum, and will not be easily discernible on the nebulous background.

I have chosen m_π (max.) as the characteristic magnitude of the

* Since this was written, Dr. Hubble has announced the discovery of Cepheids in M31, M33, M81, and N.G.C. 2403.

Novæ, because the absolute magnitude seems to be more or less constant at maximum, but varies considerably at minimum. To illustrate that statement we cite the following circumstances.

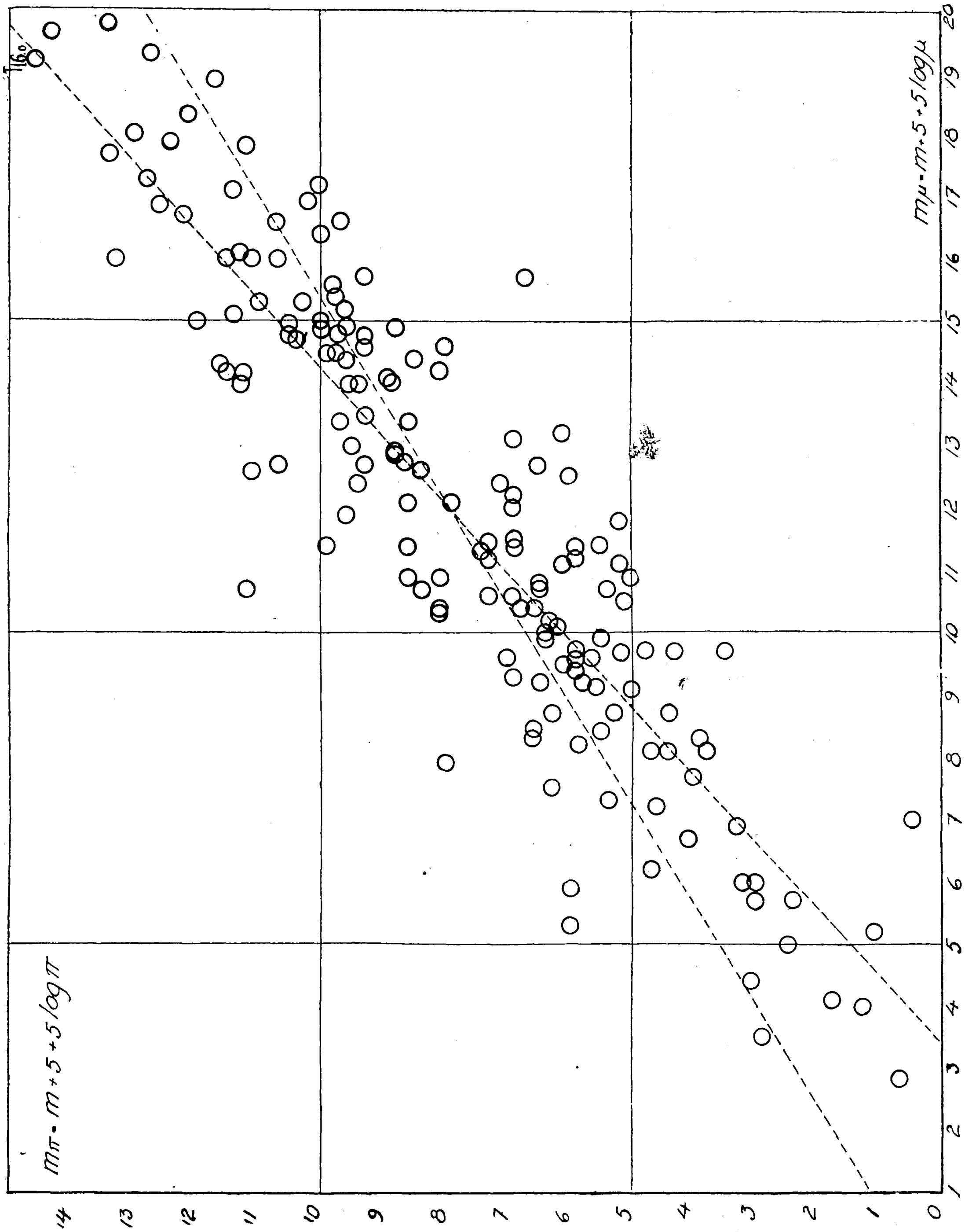


FIG. 9.—Relation between Absolute Magnitude, m_π, and Absolute Proper Motion Magnitude, m_μ, as derived from the Stars having Parallaxes larger than 0″·100.

The parallaxes of the objects *Nova Persei* 2, *Nova Aquilæ* 3, and *Nova T Coronæ* have been determined by several independent methods, and are thought to be fairly accurate. We have for m_π (max.) $-5{\cdot}4$, $-8{\cdot}7$, $-7{\cdot}1$, and for m_π (min.) $+8{\cdot}8$, $+3{\cdot}2$, and $+0{\cdot}6$. Thus

the minimum or the original magnitude varies considerably more than does the maximum magnitude, which is also seen from the table of individual parallaxes of Novæ.

In the central part of our galactic system 16 Novæ have been found. As the region in question is very far away we can take the apparent magnitudes of the Novæ as also giving the distribution of the absolute magnitudes. Excluding Kepler's Nova, which is of a singular type, we find the dispersion about the mean to be $\pm 1 \cdot 52$ magnitudes.

The 22 Novæ observed in the Andromeda nebula have, with the exception of S Andromedæ, a fairly constant absolute magnitude. Besides, we ought to remember that it is very seldom that conditions permit us to observe an Andromeda Nova at maximum brightness, a fact that increases the dispersion in absolute magnitude. The same remark refers, of course, to the faint Novæ in the Milky Way.

Computing m_μ (max.) for 11 Novæ the dispersion is $\pm 2 \cdot 31$, whereas the dispersion in m_μ (min.) is considerably larger, $\pm 3 \cdot 31$. As we have approximately $m_\pi = a + 0 \cdot 8 m_\mu$ and $\frac{dm_\pi}{m_\pi} = 0 \cdot 8 \frac{dm_\mu}{m_\mu}$, the dispersions in m_π (max.) and m_π (min.) are $\pm 1 \cdot 78$ and $\pm 2 \cdot 56$ respectively.

How do we know that the spiral Novæ are of the same kind and have the same physical properties as galactic Novæ? Of course the co-ordination of the two groups is a hypothetical method, but not bolder than many other assumptions in modern astronomy. Besides, the hypothesis gains some support from the following circumstances:

1. The similarity of the light-variations. The light-curves obey the same general law of rapid increase and a more slow decrease. The curve derived from observations of S Andromedæ is very like the curve of a typical Nova.* Secondary oscillations having a period of three days are present, and the general course of the variations is that of galactic Novæ.

2. The rate of the decrease of light is in the mean $0^{m} \cdot 052$ for 9 spiral Novæ sufficiently observed. In fact the decrease after the time of maximum is in the mean of the same magnitude as in the case of galactic Novæ.

3. As far as our present knowledge goes there are certain resemblances between the spectra of galactic Novæ and those of spiral Novæ; *e.g. Nova T Coronæ* and *Nova Z Centauri* had spectra of the classes Mb and R, and *Nova N.G.C.* 6946 had a spectrum crossed by what appeared to be series of bright bands. *Nova S Andromedæ* showed emission lines in a spectrum which seems to have been of a late type, etc.

4. There is a certain resemblance between the galactic Novæ and the Andromeda Novæ with regard to their distribution in the system in which they appear. In both cases the Novæ are crowded around the central parts. As pointed out in various papers,† the Novæ in our stellar system, although confined to the galactic regions, are not found in the star clouds, but at the borders of the Milky Way or in relation to the dark and bright nebulæ. It may have some significance that the Andromeda Novæ prefer that side of the nebula where evidently

* *Kungl. Sv. Akad. Handl.*, Bd. 60, v, 1920. † *Publ. A.S.P.*, **35**, 95, 19.

the absorption is rather marked. In some cases they have also appeared in small dark "rifts" of the nebula.

5. There is every reason for the view that Novæ are not *new stars*, but originate from different classes of ordinary stars. If the Andromeda nebula were constantly watched, ten Novæ or more per year would probably be discovered. Thus the number of Novæ outbursts in the nebula must be enormous as soon as we take astronomical epochs into account. The possibility remains that the same star passes through the Nova-stage time after time, but this seems to be a rare event compared with the number of outbursts. Thus it seems fair to assume that the Andromeda nebula is a vast stellar system consisting of millions of stars, and not a comparatively small organisation such as a stellar cluster.

In order to derive the distance of the nebula, I thus assume equality as to the mean absolute maximum magnitude of the two groups. Excluding S Andromedæ, the mean apparent magnitude at maximum is $17^{m}{\cdot}03$. Now, one must remark that the recorded magnitudes do not give the maximum brightness. The nebula can be observed only now and then, and inspection shows that most of the observed Novæ are caught on their descending branches, fading with a rate of, as before said, about $0^{m}{\cdot}052$ per day. The story of galactic Novæ shows that the objects really have to be followed every night at maximum in order to get records of the highest brilliancy.

It is rather difficult to fix the correction to maximum light on account of incomplete observations of Andromeda Novæ. From the material at hand I have in different ways computed this correction at $0^{m}{\cdot}7$. Thus the real maximum brightness of the Novæ in question would be $16^{m}{\cdot}34$. Using the value of m_{π} (max.) previously derived for galactic Novæ, we obtain :

Distance of N.G.C. 224 = **1,400,000** light-years.
Dimensions ,, ,, ,, = **60,000** ,, ,, .

The condensed parts of the nebula may exercise some absorption, which makes the Novæ apparently fainter and thus increases their distances. From the material at hand we see that there can scarcely be any appreciable amount of absorption (fig. 10).

From the distance derived, we find that the Nova S Andromedæ at the maximum reached the huge magnitude of −16. One may hesitate to accept such a luminosity. I think that we have an analogous case in the famous Nova B Cassiopeiæ of 1572. In conjunction with Mr. Humason at the Mount Wilson Observatory, I tried to identify the Nova. A discussion of Tycho Brahe's extensive measures of the position of the star showed the place to be accurate within $0'{\cdot}25$. The only star close to Tycho's position that may be the lost Nova is a star of photographic magnitude 13·7 and a spectrum of the Mb class. The spectrum shows giant characteristics, and as the difference between the present magnitude and the maximum magnitude is at least $18^{m}{\cdot}7$, we find that the maximum magnitude must have been about −16. The identification may be uncertain, but the spectral types rather suggest the star has been a Nova, as the present

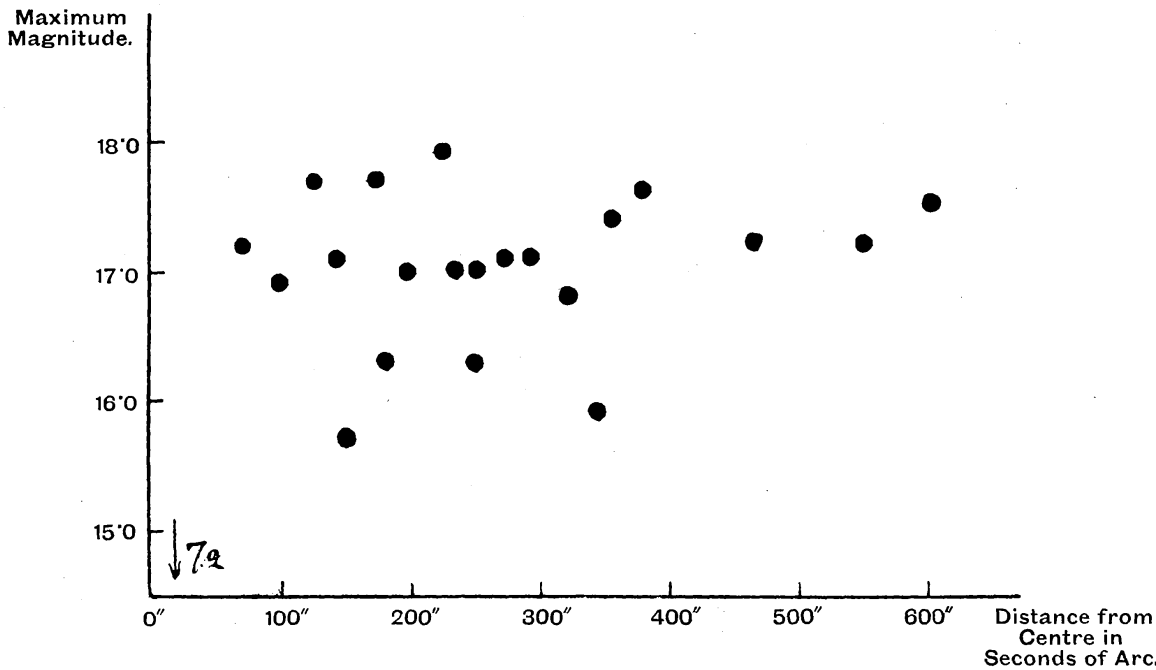


FIG. 10.—Relation between magnitude at maximum and the distance of Novæ from centre in the Andromeda Nebula.

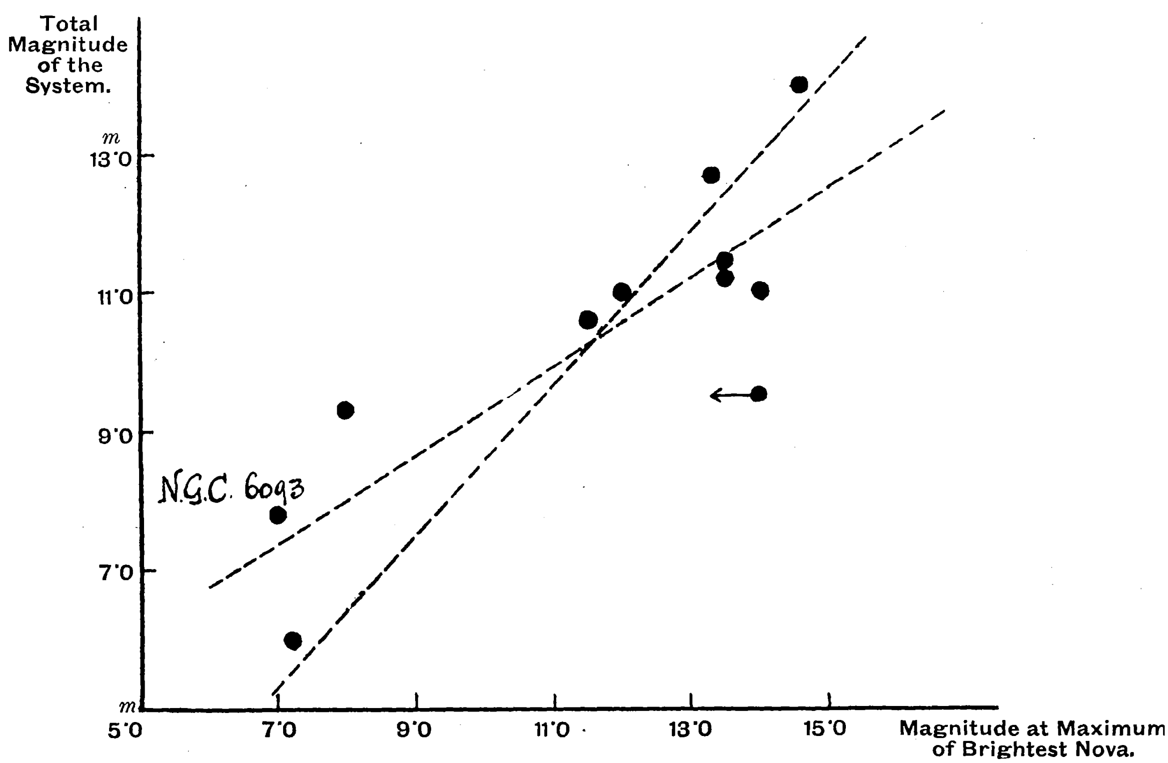


FIG. 11.—The Absolute Magnitude of "Upper Class" Novæ as compared with the Total Magnitude of the Stellar System in which they have appeared.

spectrum of Nova T Coronæ is Mb. It is true that in this case also bright lines are present, and, on the other hand, we have not found for certain any bright lines in the spectrograms of the star supposed to be Nova B Cassiopeiæ. The following table will show that neither Wolf Rayet spectrum nor bright lines are necessary features of former Novæ.

	Time since Maximum.	Spectral Class at Present.	Spectrum before Outburst.
Nova Cygni, 1920	5 years	P	
,, Aquilæ No. 3	7 ,,	P+Q	B–A
,, Geminorum No. 2	13 ,,	Q	
,, Persei No. 2	25 ,,	P	
,, T Aurigæ	33 ,,	Q	
,, Q Cygni	49 ,,	Continuous spectrum not planetary	
,, T Coronæ	59 ,,	Mb pec	
,, Ophiuchi, 1848	77 ,,	Early type; continuous spectrum not planetary.	
,, η Argus	91 ,,	F5 pec	
,, P Cygni	331 ,,	B4 pec	

An interesting analogy is the following. Supposing that B Cassiopeiæ actually reached the absolute magnitude -16, we see that this is practically the integrated absolute magnitude of the galactic system. Now, S Andromedæ was at maximum 7·2, which is of the same order of magnitude as the total or integrated magnitude, 4·9, of the nebula itself.

If we look up the cases where Novæ have appeared in spiral nebulæ we find that the same general rule will hold, as illustrated in fig. 11. The total magnitudes of the spirals in which Novæ are found have been estimated from the Franklin-Adams plates, which through the courtesy of the Astronomer Royal have been put at my disposal. The Cluster Nova T Scorpii has been included in the diagram as fitting the relation.

It is quite possible that we have to deal with two distinct classes of Novæ: one "upper class" having comparatively few members and reaching an absolute magnitude more or less equal to the absolute magnitude of the system in which they appear; one "lower class," in the mean 10 magnitudes fainter, but still reaching a very high luminosity at maximum. As the lower-class Novæ in most nebulæ other than the Andromeda will reach a maximum magnitude below 20^m, we have not been able to find the swarm of those Novæ, but have been restricted to the rather rare cases of "upper-class" objects. As pointed out by van Maanen, it is remarkable that no Nova has appeared in Messier 33,* and it is also singular that we have not seen any Nova in the Magellanic Clouds, although in this case incomplete records may explain a good deal.

But the Novæ are not restricted to spirals of the Andromeda type.

* The suspected object reported by Duncan proved to be no Nova. Hubble has signalled the discovery of a Nova in this spiral.

A Nova has appeared in the chaotic nebula N.G.C. 5253, and another one is recorded in the "globular nebula" N.G.C. 4486. The latter Nova reached a maximum magnitude of about 11·5.

The photographs of this nebula taken by Hubble show an extremely interesting phenomenon. At the border of the "nebulosity" there are numerous minute star-like points which evidently belong to the nebula, and which are not mere photographic effects. As the object is in a high galactic latitude these objects are presumably connected with the nebula. I suppose these stars to be the brightest ordinary stars that are to be found in a galactic system, and assign to them an absolute photographic magnitude of -7. As their apparent magnitude is 20, we see that the m_π (max.) of the Nova should be $-15·5$ in good agreement with the magnitude found for S Andromedæ. Using the diagram in fig. 11 and the above assumption, we find the following:

Distance	of N.G.C. 4486	=	**8,000,000** light-years.
Dimensions	,, ,, ,,	=	**6,000** ,, .

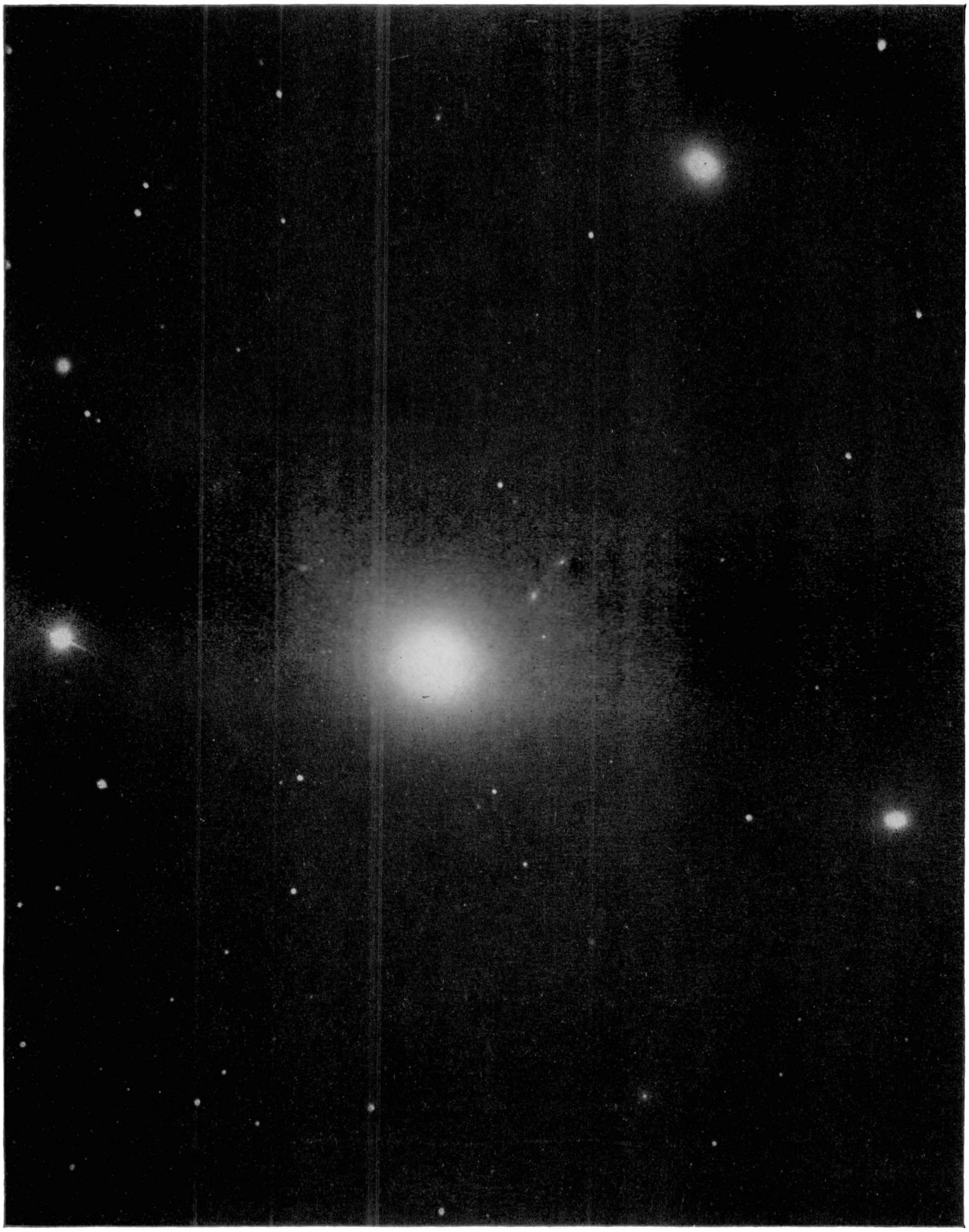
Thus the stars in this system are closely packed and the dimensions only a tenth of the dimensions of the Andromeda nebula. Anyhow, the object is too large to be comparable with a globular cluster, and it is more likely that we have in the globular nebulæ the forerunners of the Milky Way systems. It is very interesting to note that stars can be formed before the spiral arms appear.

Too much stress should not be laid on the fact that no spiral structure is seen in several "non-galactic" nebulæ. Sometimes it may only be a question of scale, in other cases, as in N.G.C. 4486, the physical constitution may be different. Still the so-called "globular nebulæ" do not seem to differ radically from the spiral nebulæ either with regard to spectral properties or radial motions. Between the typical spirals and the irregular non-galactic nebulæ, tentatively called the Magellanic Cloud type, there can be established a sequence of evolution. The spiral character probably is a feature characteristic of only a small number of the constituents of the spiral family.

(b) *Cepheids.*—Since the part of the present paper discussing the Novæ was written the discovery of Cepheids in the Andromeda nebula as well as in other spirals has been announced. Using Shapley's mean absolute magnitude of galactic Cepheids, Hubble derives 930,000 light-years as the distance of the Andromeda nebula. As Shapley's parallaxes are based on the proper motions of Boss, the question arises if the systematic correction of $+0''·013 \cos\delta$ according to Kapteyn's suggestion has to be applied to the μ_δ's or not. In an extensive investigation Ralph E. Wilson has given the proper motions of 84 objects presumably belonging to the class of Cepheids, and discussed the mean parallax of these stars.*

For 51 objects in his list having a period larger than 1·9 days, there is no doubt as to the Cepheid nature. The proper motions have been used in the following way:—

* *A.J.*, **35**, 35, 1923.



"Globular" Nebula Messier 87 (N.G.C. 4486), photographed 1920 Feb. 26 with the 100-in. Reflector at Mt. Wilson.

From a convenient table computed by Strömberg the quantities:

$$a = 0{\cdot}2111[x_0 \sin \alpha - y_0 \cos \alpha];$$
$$b = 0{\cdot}2111[x_0 \cos \alpha \sin \delta + y_0 \sin \alpha \sin \delta - z_0 \cos \delta],$$

were taken and the equations of condition formed:

$$a\pi = \mu_\alpha \cos \delta$$
$$b\pi = \mu_\delta,$$

from which the mean parallax $\bar{\pi} = \dfrac{\Sigma(a\mu_\alpha \cos \delta + b\mu_\delta)}{\Sigma(a^2+b^2)}$ is obtained. If there is a correction μ_0 to be applied to the μ_δ's the parallax $\bar{\pi}_1$ becomes:

$$\bar{\pi}_1 = \bar{\pi} + \frac{\Sigma b\mu_0}{\Sigma(a^2+b^2)}.$$

The following mean parallaxes were obtained:

0″·00271 (from μ_α's)
0″·00302 (from μ_δ's).

The fact that the mean parallaxes are of the same order of magnitude rather suggests that the proper motions in declination are not affected by any large systematic error. The application of the correction +0″·013 cos δ will wipe out the value of the mean parallax from μ_δ, and that strengthens the conclusion reached by several astronomers that the whole effect should not be applied. The proper motions of the Cepheids allow very well the application of half the effect. Combining the results from both components of the motion, we get:

	$\bar{\pi}_1$.	
1.	0″·0029	uncorrected
2.	0″·0020	μ_δ's corrected with +0″·0065 cos δ
3.	0″·0012	,, ,, ,, +0″·0130 cos δ.

The corresponding values of the distance of the Andromeda nebula are:

1. 620,000 light-years.
2. 880,000 ,,
3. 1,500,000 ,,

Some other uncertainties may be involved in the determination of the mean parallaxes of Cepheids. We have no definite idea as to the values of the radial velocities because, what is called rad. vel. of a Cepheid is what results from the assumption that the Cepheids are binary stars. Besides, one may have to consider the fact that the "velocity of the system" has proved to be a variable quantity (*e.g.* Y Sagittarii, Polaris, S Sagittæ).

The Cepheids in the Andromeda nebula are situated in the less dense parts of the nebula rather far from the centre. Thus these stars will probably not be affected by any appreciable absorption. The distance of the nebula will be obtained with considerable accuracy as soon as the absolute magnitudes of the galactic Cepheids have been derived with higher precision than is possible for the present.

(c) *Oepik's Method.*—An equation similar to the one given earlier in this paper connecting the central mass of a spiral with the linear rotational velocity of an object at a certain angular distance from the nucleus is easily derived.* The salient point in the method is to introduce the ratio I/M, representing the emission of radiation per unit mass, as determined from our galaxy. Oepik found a value for the distance of the Andromeda nebula of 1,500,000 light-years.

Using the measures of Pease for the rotation of the well-known object N.G.C. 4594, I have computed its distance according to Oepik's method and have obtained :

Distance of N.G.C. 4594 = **56,000,000** light-years.
Dimensions „ „ **110,000** „ .

The mass of the nebula is equal to $3 \cdot 10^{10} \odot$.

(d) *Jeans' Method.*—This method is based upon a theoretical calculation according to Jeans' theory of spiral nebulæ of the mean distance between adjacent nuclei in the spiral arms. The comparison between the theoretical distances and the mean apparent separation of nuclei as estimated on photographs then gives the parallax. As pointed out before,† one can take the theory for granted, but question the estimated distances between the nuclei. The so-called nebulous background in the spiral arms is as a rule certainly not nebulosity, but the light from numerous stars apparently gathered together on account of the vast distance from which they are seen. There is no reason to believe that the 100-inch reflector, although a huge instrument, has the power to completely resolve the spiral arms into separate stars. Jeans' distances for spiral nebulæ show that these objects are not closer, but may very well be farther away. As I see this question, Jeans' theory for the formation of stars does not prevent the spirals being extended stellar systems, and the truth of the theory is not affected by the results here given concerning the distances of spirals.

Another difficulty in using Jeans' method is that the computed distances between the stars are confined to regions close to the nucleus where we cannot estimate the apparent separation at all. The condensations move farther apart from each other as they go outwards, and the mean distance varies considerably with the distance from the nucleus.

In a recent paper ‡ Jeans connects his method with the relation between luminosity and mass.

(e) *Counts of Stars and Estimates of Maximum Luminosity.*—In 1920 I used the swarms of " stars " shown on Mount Wilson photographs of Messier 33 for a determination of the distance. The simple assumption was that the resolved " stars " in the spirals had an absolute magnitude equal to the absolute maximum magnitude of stars in the galactic system. This quantity was supposed to be −7, which also is in harmony with recent determinations. The value thus assigned to Messier 33 was 1,500,000 light-years.§

* *Aph. J.*, **55**, 409, 1922.
† *M.N.*, **84**, 747, 1919.
‡ *M.N.*, **85**, 531, 1925.
§ *Publ. A.S.P.*, **33**, 324, 1921.

That the star-like points in Messier 33 really are stars is proved by several slitless spectrograms taken by me using the Crossley reflector. The results of the study of these can be summed up in the statement that the so-called Nebula is composed of stars, clusters of stars, and also of nebulosity in more or less the same way as our Milky Way system.

The bright and dark nebulosities in Messier 33 and other spirals indicate other possibilities for determination of distances. In connection with a study of the corresponding formations in the Milky Way the distance of the spiral will also be discussed.

The star counts in Messier 33 permit us to fix the upper branch of the luminosity curve. Results which space does not permit us to give in detail of preliminary investigations of the resolved stars, the clusters, and the nebulous matter in Messier 33 yield distances for that nebula varying between 500,000 and 1,500,000 light-years.

Lastly, I discuss briefly the question as to the extension of the Universe.

In 1922 Dr. Charlier investigated whether an infinite world could exist according to the conceptions of Lambert.* He met the well-known objections of Olbers and Seeliger by showing that the total luminosity and the gravitational attraction become finite if a certain condition is fulfilled. A shortened proof will be given here.

Suppose that:

N_1 stars form together a galaxy G_1,
N_2 galaxies form together a galaxy G_2,

and so forth.

The individual systems are supposed to be spherical and uniformly distributed within the system of higher order; R_0 denotes the radius of a star and R_i the radius of a galaxy of the i^{th} order, which has a total mass M_i. M_0 is the mass of a star.

We then have:

$$M_i = N_i M_{i-1} = M_0 \prod_{i=1}^{\infty} N_i,$$

with the conditions:

$$\text{total attraction} = \sum_{i=1}^{\infty} \frac{M_i}{R_i^2} \neq \infty,$$

$$\text{total light} = \sum_{i=1}^{\infty} \frac{h_i}{R_i^2} \neq \infty.\dagger$$

The observer is supposed to be situated at the centre of each galaxy.

The first series converges if:

$$\frac{M_{i-1}}{R_{i-1}^2} > \frac{M_i}{R_i^2}, \quad \text{or} \quad \frac{R_i^2}{R_{i-1}^2} > \frac{M_i}{M_{i-1}}, \quad \text{or} \quad \frac{R_i}{R_{i-1}} > N_i^{\frac{1}{2}}.$$

The second series converges if:

$$\frac{h_i}{R_i^2} > \frac{h_{i-1}}{R_{i-1}^2}, \quad \text{or} \quad \frac{h_i}{h_{i-1}} > \frac{R_i^2}{R_{i-1}^2} > N_i.$$

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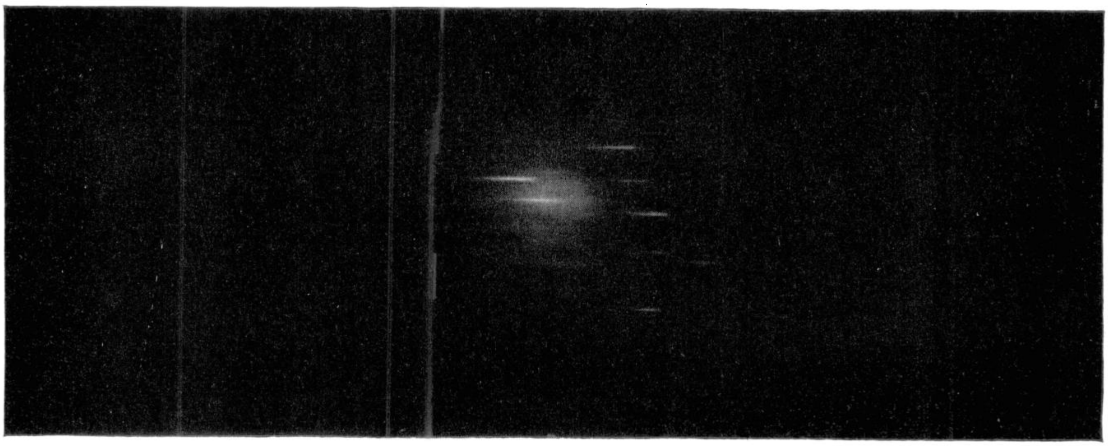
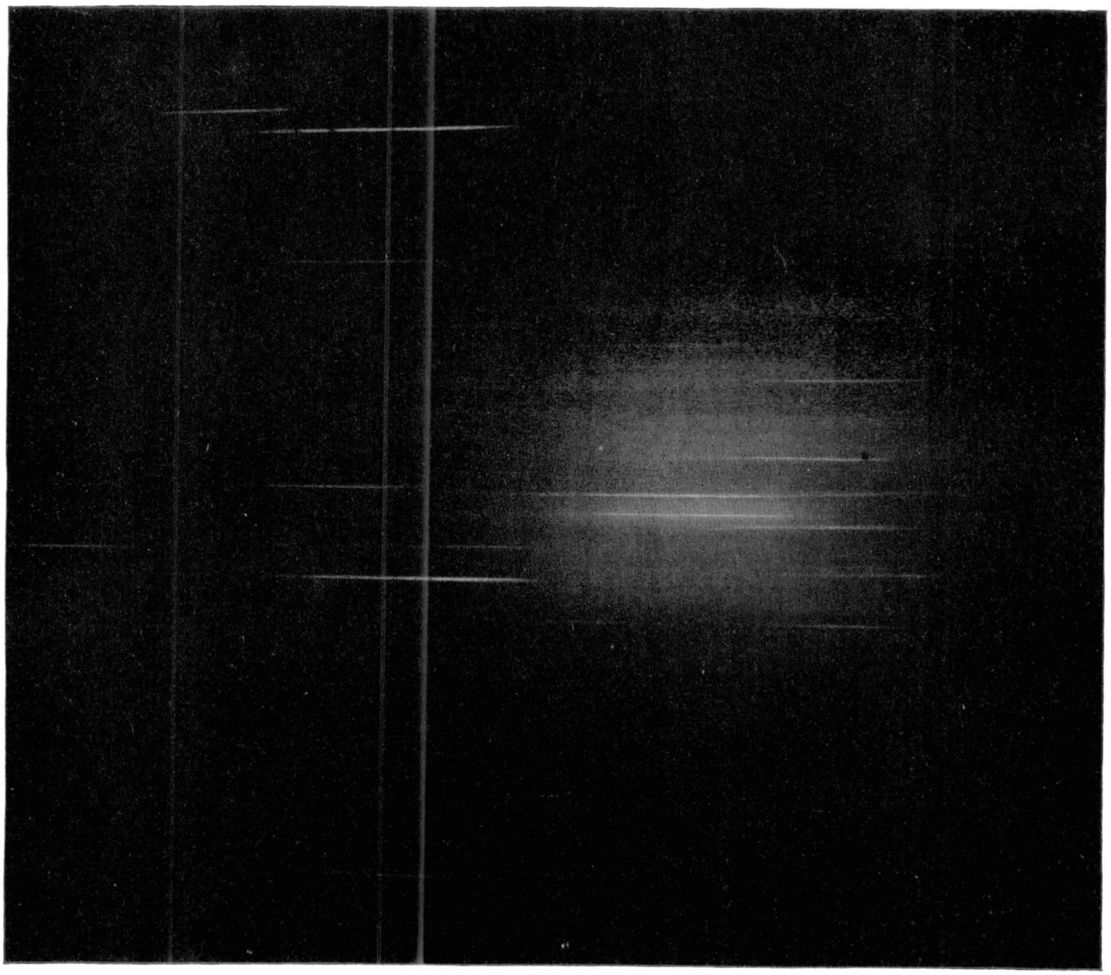
* *Medd. fr. Lund's Astr. Observatorium,* No. 98, 1922.

† According to well-known theorems in Celestial Mechanics.

I.



II.



I. Total Spectrum of the Central Region of the Spiral Nebula M 33, taken with the Crossley Reflector and Keeler's quartz-spectrograph, 1921 Oct. 29. Exposure, 8 hours.

II. Total Spectrum of the Central Region of the Spiral Nebula M 33, taken with the Crossley Reflector and large quartz-spectrograph, 1921 Nov. 2, 4, and 5. Exposure, 30 hours.

Thus if $\frac{R_i}{R_{i-1}} > N_i^{\frac{1}{2}}$ the sum of either series is finite.

As we certainly have absorption of light within the different galaxies a more general expression for the total light will be :

$$\sum_{i=1}^{\infty} \frac{h_i}{R_i^2} e^{-\lambda_i R_i},$$

and hence the convergence criterion :

$$\frac{R_i}{R_{i-1}} > N_i^{\frac{1}{2}} e^{\frac{\lambda}{2} R_i},$$

assuming $\lambda = \lambda_i = \lambda_{i-1}$, etc., and R_i large in comparison with R_{i-1}, etc.

This criterion further will be a little modified on account of an extinction effect. If we have an infinite number of galaxies in a certain direction, part of the light from the individuals will be extinguished.

Charlier identifies the spirals with the galaxies of the second order. He finds that the nearest spiral nebula must have an angular diameter amounting at the most to $5^\circ\cdot 7$. As the angular diameter of the Andromeda nebula is 2°, he concludes that the inequality may nearly be altered to the equality $\frac{R_2}{R_1} = N_2^{\frac{1}{2}}$.

Thus he finds :

Distance of N.G.C. **224** = **28** × *Diameter of galactic system.*

Supposing we know the place of our Sun with regard to the centre of the galactic system, we can easily, by assuming the galactic and the Andromeda Novæ to have equal absolute magnitudes, express the distance of the Andromeda nebula in terms of the diameter of the galactic system. Thus I have found :

Distance of N.G.C. **224** = **32** × *Diameter of galactic system.*

The good agreement between the two results obtained from suppositions so different is rather remarkable, and cannot very well be due to chance.

A comparison has also been made between the mean angular dimensions of the spirals and the mean angular distance between them. Mainly in accordance with a derivation by Pôlya,* we compute the mean angular distance between n points scattered at random on a sphere having the surface 4π.

The probability that a point is situated within a certain area may be taken as proportional to the area. The probability that a point is farther away from one selected as origin than the spherical distance α is :

$$\frac{2\pi(1+\cos\alpha)}{4\pi} = \cos^2\frac{\alpha}{2}.$$

* *Astr. Nachr.*, **208**, 175, 19.

The probability that a point is farther away than $a+da$ is:

$$\frac{\cos^2(a+da)}{2}.$$

Hence the probability that it is farther away than a, but closer than $a+da$ is:

$$-d\cos^2\frac{a}{2}.$$

The probability that $n-1$ points are at a distance from the origin between a and $a+da$ is:

$$-d\cos^{2(n-1)}\frac{a}{2}.$$

The average value $\bar{a}$ is

$$=-\int_0^\pi a d\cos^{2(n-1)}\frac{a}{2}=\int_0^\pi\cos^{2(n-1)}\tfrac{1}{2}a da=2\int_0^{\frac{\pi}{2}}\cos^{2(n-1)}da=\tfrac{1}{2}\sqrt{\pi}\frac{\Gamma\left(\frac{2n+1}{2}\right)}{\Gamma(n)}.$$

Applying Stirling's formula for $n!$ we obtain:

$$\bar{a}=\sqrt{\frac{\pi}{n-1}}.$$

Using the material in the N.G.C. outside the Milky Way, and assuming the following numerical values of the Herschelian symbols—

3″–4″,	10″–12″,	20″–30″,	50″–60″,	3′–4′,	8′–10′,	20′,
*e*S,	*v*S,	S–*c*S,	*p*S–*p*L,	*c*L–L,	*v*L,	*e*L,

the mean diameter $\bar{d}$ of 10,318 nebulæ is found to be 51″·8. As $\bar{a}$ is 3070″, we find:

$$\bar{a}=59\times\bar{d}.$$

A much better agreement with Charlier's theory is obtained by using the homogeneous material collected by Fath, from plates taken with the 60-inch reflector at Mount Wilson.* His material gives:

$$\bar{a}=\mathbf{28}\times\bar{\mathbf{d}}.$$

Thus we are for the second time led to practically the same numbers as Charlier. Our present knowledge as to the space-distribution of the stars and the spirals can be summed up in the statement:

Our Stellar system and the system of spiral nebulæ are constructed according to the conceptions expressed in the Lambert-Charlier cosmogony.

Whether galaxies of the third order exist we do not know. As it was said by Pease † that the appearance of N.G.C. 3379 suggested a cluster of nebulæ, I thought at first that globular nebulæ might be galaxies of a higher order. The results concerning the spectra and velocities of these objects show that they are not exceptions among the spirals, but probably forerunners of them. Besides, as the number of spirals

* *A.J.*, **28**, 86, 1914.
† *Mount Wilson Contr.*, **186**, 17, 1920.

is of the order of 10^6, the system of spiral nebulæ or the galaxies of the third order must be smaller than 8″. Thus they are not distinguishable from small spirals. For the present it is difficult to see how it will be possible to get a criterion for recognising such systems. The velocities of the G3's will be much larger than those of the G2's, but, of course, it is doubtful if spectroscopy ever will have the means of analysing the light emitted from the galaxies of higher order than the system of spiral nebulæ.

Upsala and Greenwich:
1925.

Naked-Eye Observations of Venus.

By J. K. Fotheringham, D.Litt.

The following observations are in continuation of those published by Professor Turner in *M.N.*, **84** (1924), 770–2. All altitudes are true, refraction being disregarded.

First Appearance in Evening.

1925 May 22. S. W. H. watched near Bicester (1° 10′ W., 51° 54′ N.). Numerous patches of cloud in N.W. rendered observation difficult. Venus not seen.

23–9. Too cloudy for observation.

30. S. W. H. saw Venus about $20^h\ 25^m$ G.M.T., and pointed it out to N. B. H.

31. S. W. H. saw Venus.

J. K. F. watched at Oxford from May 21 to May 30 and June 1 to June 5, but did not see Venus.

H. G. T. reported on May 31 that he had kept a look out for Venus at Dedham, but had had no luck as the sky was clouded.

The computed altitudes for S. W. H.'s observation on May 30 are:

	Time.	Alt. of Venus.	Alt. of Sun.
	h m	°	°
Venus seen . .	20 25	+4·0	−2·4
Venus set . .	20 57·7	..	−6·06

Stellar magnitude of Venus −3·4.

The observation is less striking than that of J. H. J. at the same phase on 1923, Nov. 7, but the Sun was nearer to the horizon at the setting of Venus than it was at the setting and rising respectively at last appearance in evening and first in morning recorded in 1924.

University Observatory, Oxford:
1925 *June* 6.

Note.—J. K. F. continued watching, but did not see Venus till June 16. He did not watch on June 15. S. W. H. saw Venus on June 15 well before sunset.

1925 *June* 29.